Projectivity of Ordinary Objects:
Restoring Topology to Architecture as Void and Liminal Passage

Abstract. Architecture's phenomenologists have expressly condemned projective geometry, blaming Girard Desargues for single-handedly initiating the "instrumental attitude" that turned Western architecture away from Euclidean wholeness. Whatever the justification for this unlikely accusation, Desargues, after being misunderstood and forgotten for two hundred years, was revived by 19c. geometricians who developed his projective geometry into a panoply of non-Euclidean geometries, without which modern physics and mathematics could not have discovered. No Desargues, no relativity, no quantum physics. Even though the 19c. reinstated projective geometry as the primary form of geometry, more basic even than Euclid, it was again forgotten at the end of the 1800s. It was up to the arts to adopt it to new theories of form and time: cubists, vorticists, Dada, surrealism, and later Oulipo played out the effects of projectivity without divulging the math behind them. Architecture theorists and even (or especially) phenomenologists were enchanted by these art movements, but they manifestly refused to recognize any debt to projective geometry. Only when Lacan grounded his restoration of Freud on topology was projective geometry given a theoretical basis outside of physics and mathematics. But, because architectural phenomenology regarded psychoanalysis in general and Lacan in particular as mechanistic, the chance to understand topology was again foreclosed. This essay wishes to move beyond the impasse.

> "Man would much rather will nothingness than not will."
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> — Friedrich Nietzsche, *On the Genealogy of Morality*
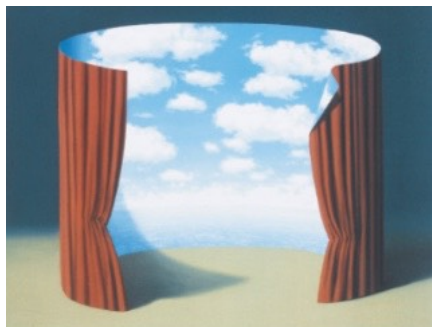
three curtains

Figure 1. Réne Magritte, *Memoirs of a Saint*, 1960. The curtain of appearance has an interrupting entry-way *and* a pretended 360° panoramic internal reality, presenting the paradox of the architectural void and liminal passage.

Here's a little mystery. There's a painting by Réne Magritte, called *Mémoires d'un Saint*, a late painting, 1960. It's a cylinder with an opening, and inside space expands to suggest that, at least when the curtains are closed again, that there is a 360° panoramic universe inside. The container/contained relationship as flipped. As with all paintings, this little segment of virtual reality invites us inside to another kind of infinity, complete with vanishing points at infinity. But, this painting, like some others, involves the clever trick of metalepsis, which is to say that it refers to itself. It is "self–intersecting" in that its subject matter, its contents, refer to its *act* of presenting contents, a bit like a theatrical version of the Cretan Paradox where, to say that he is telling the truth, the Cretan intersects himself by saying that he's telling a lie. Well, doesn't all art tell a lie, a fiction, in order to tell a truth, a truth larger than the normal ones we try to tell when we're trying to be truthful. This is a truth *about* truth, and we can only get it from two places, art and another one I'll take up later.

Commentary on this painting rarely goes further than describing its literal appearance, saying it's "like this" or "like that." Art curators' commentaries are not good sources of theory. However, most note that the curtain is an obvious sign that we're looking at a kind of stage, where the

frame has the specific functionality of an off / on switch. Even when it's parted just a bit, there is a definite idea that there is a binary of what's behind it and those on the outside put in the position of wanting to see. A circular curtain constitutes a special case, however. It supports the idea of the sky inside, that it's panoramic, and that the chance to see behind the curtain is not just an extension of our viewing space into a few more feet or meters of the same *kind* of space, but a wholly different kind of space, something more like a fully equipped world where we have a full range of possibilities. Once inside, we are allowed to go anywhere, look in any direction, forget about where we came from.

This corresponds to the illusion we normally accord to all of the "diegetic" stories we see on the cinema screen. We see them *via* a "fourth wall" that was formerly occupied by the camera and other filming equipment, but is now the position taken up by the audience. The audience exists — it's implied by the fact that the screen is showing it something — but it pretends it doesn't exist. It has an angle of view defined by the size of the screen, but as a corollary of its own non-existence, it pretends that this is zero, less than zero, actually, since when the imagination goes inside the diegetic space, there's not even a thin line left behind. the $<0°$ makes the $360°$ on the inside $>360°$, a kind of super–circularity or super–sphericity. A sphere that's more than a sphere, equivalent paradoxically to a circle or sphere with a gap.

It's more than a sphere or circle because it's us who are inside, on a *speculative visitor visa*, so the reason the sphere is more than a sphere is that we are haunting the space. Like all ghosts, we are free to float around anywhere, pass through walls, without any linear time restrictions, dinner plans, need to pee or pay taxes. Our disembodiment is simultaneously a re-embodiment, a re-positioning, thanks to this vector that slips through the curtain into a super–spherical world. A projective world, where all lines, even parallel ones, meet. As the sky indicates, the horizon is there, too; but its infinity has been brought within reach. The sun and stars are now little playthings, as small as they look and completely friendly.

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Personal statement: I wanted to show an image of Magritte's *Memoirs of a Saint* because I had been working on a way to describe projective geometry to an audience that was as non-mathematically minded as I have always been, to show that projective geometry is something we need to think about if we are in the business of dealing with the spaces and times related to architecture, but also art, literature, dreams, and the imagination in general. Before knowing about this painting, even thought I must have seen it when I visited the Munil collection in Houston nearly 15 years ago, I came up with a diagram that I thought might work. It was a cylinder with a slit, the outside I had labeled "appearance," the inside was called "the truth of truth." The inside was one way of showing projective geometry, which is normally depicted as a sphere, but if you add gravity you can talk about the way architectural voids are always coupled with a liminal passageway connecting local space, usually conceived in Euclidean terms, with the interior
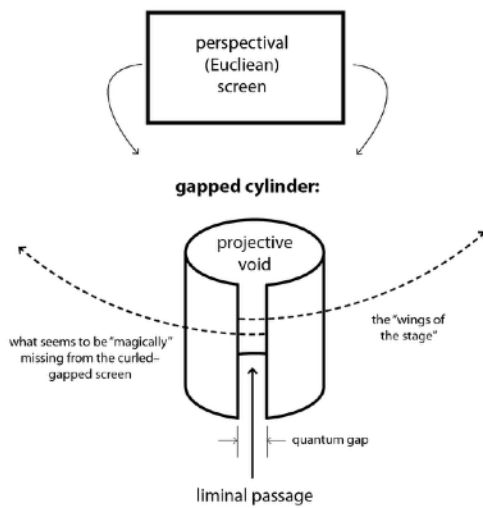
Figure 2. It is a cultural convention to consider a screen as entry into a fictive space that extends beyond the margins, while the "fourth wall" entry facing the audience is theoretically zero. Within the fictional realm, there is "panoptical accessibility." The visitor cannot see the fourth wall, but occupies a "projective void" facing in every direction. (A sphere would be more appropriate to depict this.) But, viewers in fact never really leave their literal positions in the "auditorium." Their occupancy of the work uses a liminal passage through a small slit in the projective cylinder. If the screen represents the full 360° of fictional virtuality, right and left sides are "spookily" joined. What departs on the right reappears on the left.

of the void, which is what I will argue is not just one of the aspects of architecture but its essence. You can't have architecture without the void, and wherever you have a void, you have an architecture, with or without a material building around it. As Eric Ericson demonstrated in his study of the spirituality of Northwest indigenous peoples, the void can be assigned attachments to ordinary objects: stones, trees, streams. All voids have the aura of an unmarked grave. The absent mark can be just as effective as the present mark.

I didn't want to call my diagram a diagram, because many people tend to read diagrams as reductions. I wanted to call it a *figure*, after the musical idea of something that combines a repeated motif, an embellishment that the musician can vary with every new performance, and a kind of "face" created by the music to show its character to the audience. If you accept this characterization, you'll see how my "figure" is related to Magritte's painting, also a *figure*.

There are always two things involved when we see a painter or writer say something that unexpectedly connects with some big idea. (1) The Big Idea in this case is that of projectivity. We in the architecture world don't seem to know much about this given the preference of contemporary architecture phenomenologists for Euclid over Desargues, the historical founder of projective geometry, but historians of mathematics are in agreement that projective geometry is the foundation and logical basis of *all* geometry; that it is more fundamental, in fact, than Euclidean geometry, which can be shown to derive *from* it. Projective geometry is a geometry without set distances or angles. In projective geometry, all lines intersect, but parallel lines intersect on a special line, which is called infinity and can be drawn as a circle or ellipse around any space of representation.[1] (2) The other thing, equally important: "What" is

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[1] Actually, any bounding line will do. The name of this generic separator between an inside and outside in 2-d is called the Jordan Curve. In a 3-d situation, this becomes a sphere between a viewer's Euclidean locality and an interior projective space. Like Magritte's painting, the outside of the sphere represents the function of the viewer's "compression" of appearance into a two-dimensional feature, such as a curtain, which conceals a fully dimensional interior, hence Magritte's depiction of the interior as a microcosm, miraculously larger than the space outside that seems to contain it.

rather surprising — projective geometry is not widely discussed in literature on Magritte. The "how" and "why" deserve equal attention. Magritte not only gives us a fairly accurate depiction of projectivity — this makes it a "figure" in the musical terms I wanted to suggest — but he seems in this case to be "the man who knew too much," like the fellow in the Hitchcock film of the same name who was being pursued because he had information but didn't know what or how he had it. Like projective geometry, we have another term that art/architecture criticism doesn't use very often, *kenosis.* This is when we know but we don't know that we know. Another name for this kind of knowledge is the Unconscious. We have an Unconscious, Freud argued, because we have language.[2] If, like psychotics, our status as subjects within language is damaged or foreclosed altogether, we cease to have an Unconscious. Only neurotics and perverts can have *kenosis*, so it's only as neurotics or perverts that we can call this painting a picture of our unconscious or, better, a *figure* of it; or — even better — our actual Unconscious. We can have it this way because Magritte himself had it this way. The Unconscious is shareable. This is one of the astounding things that makes psychoanalysis different from psychology, and the one thing that makes psychoanalysis congruent with projective geometry.

The simplistic diagram I was working on (Fig. 2), which I was hoping to make into a figure, tried to show how the architecture of the theater facilitated the emergence of a projective geometry, namely the void on the stage that allows the audience to fictionalize its passage out from the darkened auditorium into the imaginary space created by the dramatic act. Architecture plays the crucial role. The physical building diagrammatically divides the space into stage and auditorium, schematically simplified by dividing a circle into two 180º parts with the diameter of the curtain, which works like an off/on switch. Curtain down, the auditorium is powered up, lights on. Curtain up, the stage is lit, the auditorium goes dark.

But, this binary alternation between stage and auditorium is not what happens if the drama on stage succeeds in engaging the audience. The audience must imagine that is *inside* the drama, and in cinema this is even more obvious because the camera's extensive mobility simulates the experience of a dream. Film editing can transport us to scenes that are far distant from each other, convincing us that we are *inside* something we won't get *outside* of until the end of the story. The story is the sphere separating the Euclidean auditorium from the projective space–time of

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[2] Architecture's investments in psychology over psychoanalysis mean that the Unconscious is discounted or reduced to the function of a latent or pre-conscious awareness. The Freudian–Lacanian field, in contrast, shows how the Unconscious is a function not of the Imaginary, which Jungians would prefer, but of the Symbolic — networks of symbolic relationships as well as literal language, where gaps and defects in signifying chains require that subject, to be subjects, will be brought within the Symbolic but radically mis-recognized. Alienated by the very thing that pretends to bestow identity, the Subject ($ for Lacan, to indicate that the Subject is barred/divided by its conscious and unconscious nature) has no utopian future, as architectural phenomenology seems to promise it.

fiction. From the outside, the sphere encloses a void; nothing. From the inside, the void is anything, everything. Our access to the interior is a liminal passageway that is less than nothing.

Magritte *must have been thinking* about this theater situation in terms of projectivity, otherwise why would he specifically show a curtain and a sky horizon? Magritte has become a geometry teacher, able to describe a complex subject to an audience who has no prior familiarity with projective geometry. How did this happen? Magritte was interested or even we would say obsessed with the fourth wall status of painting: the a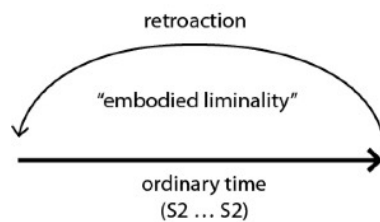bility of painting, as a *surface* to also be a passageway. His question always seems to be "A passageway to *what?*" Once inside the fictional world of his realistic representations, it seems that we are in a world of pronouns. They are not specific things but generic things. Not this object or that object but any object, every object. Pronouns are not points, they are vectors. They point to somewhere else. And, following a rule of projective geometry, they are a member of a family of parallel lines that meet at a horizon ("infinity"), a vanishing point that always appears with its twin, an antipode marking the other end, a negative, of infinity. The pronoun–vector's spatial presence is a temporality, pointing to a specific time, a future anterior, a time *by the time of which* we will know what they are, a case of *aprés coup*, of retroactive knowing: "You'll know it when it happens." This temporal vanishing point is at infinity thanks to its retroactive structure (Fig. 3). Like the beginning of a sentence that is not understood until the end of the sentence, the ending "vanishes" because its antipode, the beginning, is simultaneously present.

Figure 3. Retroaction is the temporality of *kenosis* structured by chains of signifiers (S2 … S2) within a local/Euclidean context. Embodied liminality is the space beneath the curve of retroaction in the sense that the reverse movement is never fully coincident with the forward movement.

This is another quality of the Unconscious, what I am calling *kenosis*. You don't know it now, but you will know it in the future, and when you know it you will know it only because, *aprés coup*, you won't have to ask. You know it when you see it. You know it because it has connected you to something in the past that authenticates it. True authentication alway involves this retroaction, this *aprés coup*, action. In *kenosis* you have something all along, but it's hidden. You don't know it until a point in the future when this thing you possess is connected to your first possession of it, when you got it but didn't know what you were getting, or that you were going to keep it as long as you didn't realize you had it. You could draw another figure here, it would be a line going straight along from past to future, a kind of Euclidean line. It would end at a point where another line arcs backwards to the start of the Euclidean line (Fig. 3).

Figure 3 is a picture, a figure, of the way we look, from our position in Euclidean space, our "locality," at projective space, which seems to be curved in some way. If we complete the idea of curvature into a figure, a sphere, we can model what happens inside the same way we would

finish off Magritte's painting of the cylindrical curtain. It would have a horizon; it would have vanishing points. Every set of parallel lines would have *two* vanishing points, one in front of us, another antipodal to that. The auditorium's semi-circular share of the theater building would vanish, shrinking to less than nothing, <0º, and the space inside would super–size itself, to be greater than a its assigned space, >360º and we would now remember what Pascal said, that God was an infinite sphere with circumference nowhere. Our point of view inside the sphere is as ghosts inside the fictional space–time of art, with free access to any and all points. The center of God in Pascal's projective sphere, like that of projective space, is everywhere.

Leibniz and Newton had a debate about the edge of the universe. Newton, good Euclidean that he was, argued that that the physical universe had to be finite, so there had to be an edge. Leibniz argued that any line would require the existence of space on the other side of it, which would be unthinkable. Resolution of this problem didn't come until Einstein came up with his own figure, a universe that was finite and unbounded, *curved* but also *closed*. Einstein's infinite sphere is roughly the same as Pascal's, and both of them are projective, not Euclidean. It would be hard at this point to say that projective geometry is an irrelevant idea meaningful only to mathematicians. It's the space we actually live in, although Euclidean locality is required, if only to provide liminal passage with a departure point.[3] The quantum effects observed at the cosmic scale are, for us on earth, the substantial small–scale experiences of the uncanny. Both "quantums" break the rules of Euclid. We cannot define a "homey" Euclidean world and forget about projective geometry. It is not just the ground of all geometry, as mathematical historians have demonstrated, it is the basis of the uncanny that makes architecture's essence, the void and the liminal passage to the void, what it is, i. e., architecture and not just building. Or, more enigmatically, architecture as the Unconscious, the *aprés coup* of building.

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Now we come to a mystery story I've been running down for the past three years or so, about why architectural historians, going against a lot of other historians, still choose to misrepresent or even condemn projective geometry, blaming it for all our present woes. This is like what the Soviets used to do with photographs when a power figure fell out of favor. A photo originally

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[3] Another issue about Euclidean locality: A feature of projective surfaces is that they are non-orientable. There is no up, down; no north, east, south, or west. This does not make them "anti-humanistic" in that cardinality is a cultural commonplace. Rather, it shows precisely how non-orientability produces anxiety that is remedied, ritually and sacrificially in early cultures, to *re-establish* orientation without fully disengaging from the anxiety that energizes human interest in maintaining it. One proof of this is the way that the walls of ancient cities were doubled, requiring to maintain a religious space between an internal and external wall that accommodated periodic ritual renewal. Sacred calendars are structured around the staging of renewals where anxiety is re-introduced in order to be re-situated. The non-orientability of projective (self–intersecting) figures such as the Möbius band, cross–cap, and Klein bottle makes them good models of how anxiety is tied to projectivity by negating Euclidean locality.

showing a line of dignitaries would suddenly have a blank spot. Architecture's blank spot is projective geometry. Historians, even the respected authority on artistic perspective Hubert Damisch, praise the inventor of projective geometry, universally agreed to be Girard Desargues in the mid–1600s. To make his point, Damisch quotes Michel Serres, in his book *Le Système de Leibniz et ses modèles mathématiques* (p. 693):

> Euclidean geometry posits a homogeneous space in which all points are equivalent, in which all points are of no account, as regards spatial composition … The geometry of Desargues, by contrast, posits a space organized in relation to a point of view through which order is imposed on a random variety of the first. Here the point encompasses space and space encompasses the point, the word "encompasses" being understood to embrace not only geometry, but vision and thought as well."

Desargues was aided by no less a figure than the young genius, Blaise Pascal. But architecture historians have rejected Desargues because, they claim, he single–handedly caused architecture to turn away from the Euclidean assurances of the subject's place in a representable world and toward the mechanistic attitude that created the Industrial Revolution and the crisis of modern science.[4] On the face of it, it would seem hard to prove that Desargues, who was completely misunderstood in his own day and forgotten until the nineteenth century, could have been responsible for anything that earth–shaking. Desargues' architectural motive seems to have been, innocently, to make it possible to create perspective drawings without having vanishing points falling far off the drawing board. His other project was to make it possible for architects to give stonemasons accurate descriptions of the complex surfaces they needed to construct large vaults. I failed to be convinced of the pure evil of Desargues' motives. Yes, stone masons would resist having their secrets given away by documents that could be worked up by any trained draughtsman. It was also evident that Desargues didn't think through his PR problem when he changed the names for all of the elements of perspective drawing. But, these seem to be misdemeanors, not felonies.

What puzzled me was the zeal against Desargues, which seemed to relieve the historians from having to actually describe what projective geometry was, so that we might decide for ourselves. In a way, we already knew projectivity without being told the name. It was the legacy of cubism, Surrealism, Dada, and other artistic breaks from representational realism made around the beginning of the twentieth century. Although it is bad idea to approach projective

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[4] One early dissertation at Virginia Tech put it this way: "Desargues was present at the birth of mechanism — the world picture upon which experimental science was founded. Desargues was a mechanist at a time when there was no better way to make enemies. The life and work of Desargues can help us understand the birth of mechanism."

geometry in terms of a "fourth dimension," as Duchamp tried to do, the end result is the same.[5] Time must be re-introduced to the static Euclidean field, and so cinema becomes an integral part of modern painting and architecture. Vision's natural desire to flatten appearances became a compression squeezing surfaces free of ornament and into a flat surface, painted white or some other primary color, "coded" as if to say that surfaces separated in Euclidean space were to be joined back together in the projective imaginary.

How are we to treat the legacy of modern art without understanding projective geometry? The impressive introduction to the revolution of Western thought around perspective theory given by Hubert Damisch makes an important miss-step in my view. Damisch looks to the history of perspective to find possible antecedents for Desargues' revolutionary discovery of projectivity. Why didn't he think to look to the history of geometry rather than perspective? Had he done so, he would have immediately found something of interest.

In the mathematical origins of projective geometry, figures and surfaces do not require and in fact do not support measurement. This is important in light of architecture's claim that Desargues' projective geometry is directly responsible for the virtual realities represented by computer graphics. Everything in virtual reality, in fact, requires Euclidean measurement. The ability of virtual reality to represent topologies at all is based on their ability to use algorithms that continually adjust *translations* made with specific and precise measurements of distances and angles. Topology cannot be directly represented by computer graphics. It must be animated and approximated by graphics constructed as Euclidean vectors. Thus, the famous "bottom line" of architectural historians condemning projective geometry *via* Girard Desargues cannot be supported. This is not intended as a simple refutation. Rather, I would use it as an excuse to side with skeptics of so–called parametric procedures of computer–generated "virtual realities" and invert their term, "virtual reality," into a Nemesis and antidote, "the reality of the virtual." This has been done by Slavoj Žižek already, so I am completely unoriginal, but I would like to put a hold for the moment on Žižek's references to Lacan's thinking, to show off the general validity of this word–palindrome.[6]

The "reality of the virtual" assumes that, in all perceptual experience, the sensing subject constructs what it takes to be *given* appearances. The subject's active role is suppressed. We

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[5] Marcel Duchamp's analogy of projectivity was based on the analogy of 3-d operations to 2-d ones. It is easy to understand moving a planar figure through a third dimension and then applying the same idea of movement to the case of a solid object moving through a fourth dimension, but the analogy gives the impression that the fourth dimension is a convenient fiction. Projectivity offers the alternative of a space where infinities can be algebraically described, tested, and confirmed. Just as experiences of the uncanny *as such* are concrete universals to be found in all cultures at all stages of development, projectivity is not *like* something more real, it is the *very model and basis* of the real.

[6] Slavoj Žižek, *The Reality of the Virtual* [podcast], Ben Wright, director, 2003. URL:https//zizek.uk/slavoj-zizek-thereality-of-the-virtual-2004/.

don't say that, in perceiving, we impose our own ideas on to passive objects, that are no more than mirrors of our pre-conceptions. We allow that appearances "belong" in some way to the objects that appear to lie behind them. Our subjective influence amounts to a kind of "compression" of appearances into a 2-d format that suits our own retinal physiology. To see depth we must involve muscular actions that take place in time. In effect, the third dimension springs into being *after* a fourth, temporizing procedure. We focus our eyes, squeeze or lens, shift our point of view. These measures allow us to distinguish a flat cut–out from a 3–d object. Shifting position will show off any topology of a solid space, and we use the correlation of our point of view with the vanishing point on the horizon as evidence of the way perspectival space can be indexed by something lying outside of it externally and internally (our movement). Virtuality is the curtain, the curtain in turn is the product of the eye's retinal necessity of sensing things along a surface, and using "muscular" compensation (movement, squeezing the lens of the eye, adjusting the eyes' parallax position, etc.) so that we might re-order the spatial dimensions to read 1, 2, 4, and 3. The perception of three-dimensional depth requires the intervention/insertion of a fourth dimension, whether we describe this as a temporality (changing our point of view), muscular effort (focusing our eyes, parallax shifting), or waiting for things to move, the better to see figure–ground distinctions that disclose measures of depth. The 1243 sequence shows that human sense experience will always be a kind of virtuality, components of which can be culturally varied and taught, others which can be left to chance or the accidents of individuality.

This inserted fourth dimensions is assignable. Our primary 3–d virtuality is partly a product of culture, which teaches us what things *should* look like, and our personal experience, where we add and subtract to this cultural script. We accept photographs and realistic paintings as equivalencies, because, to put it plainly, it is the exchange between one acceptable virtuality and another. Differences between points of view is not just allowable but expected. Space is *refracted*, curved. We see things in dreams and know they are not "real" as much as they are "actual." In the same way, we regard appearances as a mask of truth, and never "the real thing." Western philosophy could be described as 3000–year–old discussion about this matter.

Kant put his finger on it by positing two things: (1) the supposed "thing–in–itself," an objective state of the object un-reworked by subjectivity in any way, and (2) the inevitable production of antinomies regarding the media of our subjective virtuality. Freud in a certain sense borrowed his concept of the Thing from Kant, showing it more to be a *product* of our awareness of the virtuality of our virtuality, i. e. our self–conscious admission that we play a role in the deceptions of nature.

Reality of the virtual

What then is the *reality* of the virtual that Žižek cites? Is it the thing–in–itself that seems to be covered by a curtain of appearances, "flattened" by the eye into imaginary layers or veils reveal-

Figure 4. Frontispiece, Giambattista Vico's *New Science* (1744), known as the *dipintura*. Lady Metafisica stands on top of a sphere that is an inside–out reversal, showing the cosmos with its astrological belt. The liminal passage into the sphere of the image was intended to be seen as the eye of God by the Inquisition censors, but Vico makes it clear that it is none other than the eye of the present viewer/reader of *The New Science*, who must reconstruct the text in the act of reading it. This makes the image non-orientable, i. e. fully projective. The altar's segmentation seems also to refer to a problem of projectivity, the Delian Paradox (how to double the volume of a cube), which can be done not by Euclidean logic but, surprisingly, through origami. The helmet of Hermes (lower left) is a visual lipogram: the only object in the *dipintura* that is not mentioned in Vico's otherwise complete inventory in his preface.

ing truth only by stages? Or, is it the Freudian Thing, the resistant residue of language's inability to say what it means, of the symbol's secondary (and too late) job in presenting the truth to us through perception? Žižek has a different, ingenious way of approaching this. For the "first" virtuality that is "Euclidean," local, and constructed, there is a necessary *second* virtuality that makes the first virtuality come across as effective. Žižek demonstrates this through examples given for each of Lacan's three primary domains of subjectivity, the Imaginary, the Symbolic, and the Real. Without going into the specifics of this system to spare non-Lacanians the trouble of reading up, let's say that secondary virtuality is whatever works as a kind of Aristotelian efficient cause: *it is that which makes things work*.

This is very much in line with Giambattista Vico's idea of the *factum*.[7] In fact, the majority of those who misunderstand Vico's important category of "the made," do so because either they disregard the virtuality of the human perceptual world or they see making as open to the conscious access of any and every subject/maker. Vico was clear: the human world is a construct, and therefore virtual, in the first sense. But, importantly, the construction of this virtual world remains difficult if not impossible to access. He claimed that it took him a good twenty years to reach his own theorem, the idea of a *self–intersecting*, virtuality–producing metaphor that worked *on the condition* that it remain inaccessible to the humans who

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7 Giambattista Vico, *On the Most Ancient Wisdom of the Italians*, trans. Lucia Palmer (Ithaca: Cornell University Press, 1996.) Architecture historians have it that Vico's aphorism, *verum ipsum factum*, come from his major work, *The New Science*, but in fact this phrase never occurs in that work.

used it.[8] Vico's projectivity is not just circumstantially projective in geometric terms. The frontispiece he used for his major work, *The New Science*, shows clear evidence that he understand the interior of Magritte's cylinder. Vico even constructs a liminal passage into this interior–exterior by tagging it with the helmet of Hermes, shown clearly at the bottom left of the image but mentioned nowhere in the text that he claims has covered every item. Vico knows that the token of liminal passage is the lipogram — a "missing letter" — that, like Lacan's famous letter, will always reach its destination (Fig. 4).

Secondary virtuality is evident at any scale and within any fragment of human perceptual–pictorial (primary virtual) experience. Unlike the metaphor of the fourth dimension, it is not a weird space "out there," conceptually or physically, but a property of visual/spatial experience anywhere and everywhere, a commonplace, an ubiquity. A general model can be constructed out of the appearance/reality paradigm. Appearances are always regarded as such: something that are seen to be concealing something that, at the moment, is not immediately present. This can be extended to other kinds of experience. We say "hello" to someone we see on the street; they must come from somewhere and must be on their way to somewhere else. We add this mentally, but in the role of a kind of place–holder, of things we don't need to know even though we know that they must exist. These are mostly things we don't *want* to know. A Hemingway character, for example, advises his friend not to think about the sex lives of his acquaintances, otherwise it will destroy any possibility of friendship. Other examples follow from the traditional stupid example given in Philosophy 101. You see only a few sides of a cube at single glance but are not troubled; you "know" that the other sides exist, and that if you took the trouble to see them they would be there. Their primary virtuality, which says "Look, there's a cube!" is supplemented by a second virtuality that says, basically, "Don't bother to walk around." It wouldn't be an appearance of it didn't hide other appearances.

With the work of representational art, the case is more complicated but more theoretically informative. A painting's frame contains a scene that is cut off at the margin. We imagine that the painter might have easily included a bit more or a bit less; the "reality of the virtual" is that the represented scene doesn't actually stop at frame's edge. Rather, it wraps 360° around anyone who imagines they are standing inside the scene. It would be ridiculous to think that a scene that is painted or photographed itself would stop at the edge of the frame. Primary virtuality defines what we see of the representation, a sample view; secondary virtuality is what "must be there" for the representation to be a representation *of* something. Secondary virtuality is what makes the primary virtuality work, although it is never in the main picture it makes possible.

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[8] Note that architectural phenomenologists who have attempted to appropriate Vico have failed to grasp the essential inaccessibility of the Vichian *factum* and, instead, made it a principal feature of Gadamerian/Ricoeurian hermeneutics. If Vico can be said to be a phenomenologist, it would be in the sense of Ernst Cassirer's idea of "symbolic form" — the way any "displacement" of perceptual experience into a "symbol"requires, axiomatically, a full domain in which the symbol forms a part of a whole.

I would call secondary virtuality "effectiveness through absence." To understand it in more depth, I would compare it to a rhetorical form known as the enthymeme. This is a specific kind of persuasive speech that is effective because any positive depiction of motivation or instruction is intentionally left out. A general needing to get his soldiers to fight yet another useless battle could simply say, "Go out there and fight!" Instead he/she advises them to cut their losses and go home to their wives and families. By portraying their desire for self–preservation as normal and ordinary, the general is silently saying, "But, my soldiers, you think of yourselves as extraordinary, heroic, *not normal*." The soldiers makes this conversion for themselves, silently, and the general gets the desired result.

The enthymeme can be explained as secondary virtuality by showing how its parts correspond to the standard model, the famous syllogism cited by Aristotle, "Socrates is a man, all men are mortal, therefore Socrates is mortal." The so–called middle term is man/men, used first as a set containing Socrates as a member. "Man" predicates something about the subject, Socrates. This is the predi-*cating* position. In plural form, "Men" becomes a predicate of mortality. It is the subject, mortality says something about it (the predicat-*ed* position). The switch from predicat-*ing* to prédicat-*ed* works as a kind of x and $1/x$ — the two terms cancel each other out. They are silent, meaning that they don't appear in the conclusion. Reversing the logic, going from the conclusion backwards as in the *aprés coup* of the meaning of sentences, shows how secondary virtuality works as an added element with reversal capability. It can be container or contained, primary or secondary. This capability to switch polarity allows the conclusion to work. Crisscross actions and secondary virtuality are synonymous. (Below, we will see that the crisscross is also primary to projective geometry.)

Consider what might have been Desargues' first thoughts about secondary virtuality, had he seen Slavoj Žižek's YouTube video. Looking at things in perspective, it seems obvious that parallel lines *do meet*, at a point that more or less coincides with the infinity of the horizon. The vanishing point is *what makes parallelism work* when we encounter it in everyday visual experience and in its representations of that experience. We have to abstract ourselves from this personal and concrete experience to conclude *rationally* that they really don't meet, that both lines would be perpendicular to a third line crossing them, one of the less complicated "proofs" of Euclid's Fifth Postulate. Perspective give us something true to our visual experience, but Euclid presents a contradiction. We believe that, in seeing only appearances, that our perceptions are *wrong by definition*, and that Euclid must be right. But, what if this is not the case? In what sense could we say that parallel lines actually meet somewhere?

We are not prepared to say that parallel lines meet just because we think we see them meet. We don't trust our senses for many evolutionary–biology reasons. They are often fooled. But, what if we could *prove* that there is a "projective" reality, not "somewhere, over the rainbow" as Dorothy sings in *The Wizard of Oz*, but rather a reality that is and must be present in order to
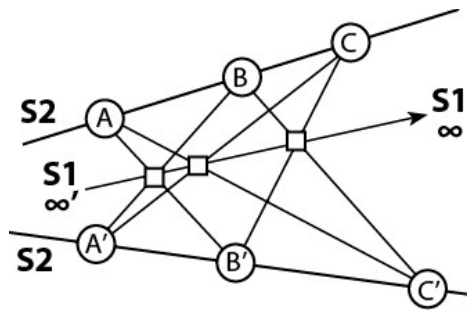
Figure 5. Pappus's theorem is easy to construct. For points lying *anywhere* (no specific distance from each other) on two lines, S2…S2 (at no specific angle to each other), crisscross lines connecting reversed pairs (AB'/A'B, etc.) cross at points that will lie on a third line (S1) that will be *projective*. That is, S1 is virtual but causal in relation to the virtuality of the presentation. Thus, it terminates in a horizon drawn around the space of representation.

make appearances work. This is a shift in the way we question the existence of projectivity, a "Vichian shift" in that, in the process of human invention, there must be from the very beginning two virtualities, not just one (an appearance, concealing something behind it), an object inaccessible to language and representation but nonetheless Real, a way of signifying (S1) that, by not being a literal signifier, is able to order all other signifiers (S2). This does away with the Kantian thing–in–itself but not with the effect of the thing–in–itself. This sees the thing in itself effective as a "positive absence." It has some properties related to authenticity (S1 really is what makes things, S2 … S2, hold together), but also some properties of an independent existence that can be confirmed through experiment and replication. In other words, secondary virtuality, unlike primary virtuality (which tries to argue that behind appearances there is something objective), is a *scientific entity* that, despite its "non-existence," can be determined, calculated, scaled, and replicated. This is probably what excited Desargues. Where did that excitement and confidence — that he had discovered something actual and not just a convention or way of seeing things — come from?

The answer — central to the theory of projective geometry, and which even those writers who have condemned it agree is central — is a theorem that is not mentioned, even by the most comprehensive texts on perspective. This is one reason Damisch's decision to pursue answers in the history of perspective rather than the history of geometry was a conceptual failure. Architectural theorists, following Damisch, have ignored this answer entirely, and do not once mention the name of its inventor. This is Pappus of Alexandria who, in 300 b.c., discovered the theorem that (1) lies behind projective geometry and, consequently, (2) behind all other theories of geometry. And, within this theorem is ("0") an ordering principle that lies behind it, rarely if ever cited even by proponents of projective geometry.

The 1, 2 … 0 of the theorem of Pappus is not complicated, but it demonstrates something profound. It shows that geometrical relationships can exist in a determinate way without the need for measurement. Euclid required a straightedge to draw lines and figures and a compass to mark out measures along these lines and figures. Pappus threw away the compass. Nothing in the projective geometry requires measurement. But, *everything* in the theorem depends on a specific way of relating figures involving a crisscross, the "zero component" of Pappus.

The theorem is easy to explain (Fig. 5). For any two lines lying on the same plane (S2 in Fig. 2), sets of six points, three on each line (ABC / A'B'C') connect in all the possible combinations to produce three intersections ("meets"): these will always be co-linear, no matter how far or close the points are to each other, and no matter at what angle the lines are to each other. But — and this is critical — the connections must "crisscross" in a precise way. The order of the points must be A on the first line to B' on the second line, then from *A'* on the second line back to B on the first line; A to C', A' to C; B to C'and B' to C. This is the same as saying that co-linear points ABC have "internal" relationships with themselves: AB, AC, BC. This internal relationship can be revealed by constructing a "shadow line" at any angle, with "shadow points" placed at any distance from each other. Spread across two lines in a plane, the internal relationship can be "stretched out" to show that its internal order can be "confirmed/revealed" by a third line, using each letter's "shadow" on the other line. AB, AC, BC thus have a kind of parallel order that materializes as this third line. The line that is A, B, and C is projective *in that* it has a shadow relation to an infinity that is internal. Like the external vanishing point, this infinity is determinate, drawable, and precise. This is the whole idea of secondary virtuality as a kind of Efficient Cause. It is the reason that things work. Like the enthymeme, it is the reasons the soldiers return to battle despite their general's discouraging words.

The crisscross means that, although the literal drawing sits on an ordinary flat plane, there is a virtual order present, revealed by giving voice to the twist that is latent within the order of points. This is a switch between a framed and framing function, just what we saw in the enthymeme's use of silence. In effect, the flat surface is treated as an origami. The surface had materially supported (= framed) the graphic lines, but it was brought into play by considering its status *as a surface* (the frame framed by itself). Strangely, there are no geometry texts that point out this origami function, which is (literally) what makes the theorem *work*. The twist is the secondary virtuality of the theorem's apparent success in showing that every two lines have a third line determined by twisting the surface on which they are inscribed.[9]

"Twisting the surface of representation" has a history in Greek mathematics. In Euclidean geometry, the limit of calculation ends with second-degree equations (involving the square root of 2). As Plato proved in *Theatetus*, "even a slave boy" can be led to find out how to double the area of a square (and, in the process, discovery the Pythagorean theorem).[10] However, to double a cube, the cube root is required, and although this cannot be discovered by use of a straight-edge and compass, it *can* be solved by folding the paper on which the graphics have been

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[9] There is a meaningful connection between this characterization and the role of origami in solving third–degree equations. See Zsuzsanna Dancso, "How to Trisect an Angle with Origami," *Numberphile*; https://www.youtube.com/watch?v=SL2lYcggGpc&list=PL20JPCodT5O2Lp4r2bLuYIi9RZA8c5yLN.

[10] The choice of a slave–boy was not accidental or lost on Plato's earliest readers. Slaves took orders; from this relationship one could deduce that the proof of doubling the area of a square would be tautological, i. e. requiring no "choice" of anything other than a mechanically available option.

drawn. *Any* third degree equation can be solved by "origami," but no third degree equation can be solved using Euclidean methods. Pappus must have been excited to find something relating to the famous "unsolvable" Delian Paradox, where to rid Athens of the plague, the Oracle required that the volume of the altar of the temple at Delos be doubled. Getting rid of the plague certainly appeals to the theme of effectiveness. Involvement of projectivity in this ancient example, though indirect, points to a definite awareness of the function of effectiveness and the medium of secondary virtuality, projective space that "folds into itself."
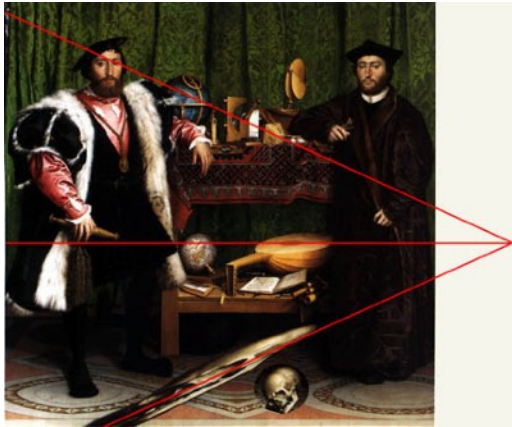

Figure 6. Hans Holbein, *The Ambassadors*, 1533. The painting's anamorphosis includes reticular lines that pass through the blurred skull image (corrected version added) and form a triangle implicating whoever finds the "sweet spot" to see the skull in correct perspective and the crucifix at the upper left, half–hidden by a curtain. The date compels us to read the painting as both a double and single surface, the *verso* acting as a projective supplement, a secondary virtuality showing how the figures on the *recto* actually work.

How could Pappus have been so forgotten that his simple–to–understand theorem, which motivated Desargues and Pascal, the heroes celebrated by historians of mathematics, *is never mentioned once*? And, how could Desargues, a true genius, be both forgotten and responsible for a catastrophic revolution, the conversion of Euclidean "harmony" into instrumentalism's mechanistic nightmares? Without Desargues, there would have been no expansion of theoretical geometry in the 19c.; no Relativity; no quantum physics. Neither would there have been cubism, Surealism, Dada, Vorticism, and all the rest.

The proof of continuity between Pappus and art's definitive responses to the potentiality of projective geometry lies in those examples where the geometries on the canvas point to actions of folding, crisscrossing, and turning inside–out. (I prefer Lacan's term, éxtimité, Englished into extimity.) But, there is proof that projectivity was known, at least in these variations on the origami theme, before the 19c. Tellingly, the (visual) examples I would cite involve anamorphosis. This suggest that, as in the case of the uncanny, folk practices and art forms can provide alternative routes to projectivity. Malden Dolar has raised an important issue.[11] Anamorphosis came into the picture primarily through Jurgis Baltrušaitis's comprehensive study, published in 1955 in French.[12] Baltrušaitis radically historicized anamorphosis, giving it a decidedly Western, Enlightenment rôle. But, as Dolar points out,

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[11] Mladen Dolar, "Anamorphosis," *S: Journal of the Circle for Lacanian Ideology Critique* 8 (2015), Capitalism and Psychoanalysis; http://www.lineofbeauty.org/index.php/S/issue/view/8.

[12] Jurgis Baltrušaitis, *Anamorphoses ou Perspective curieuses* [*Les perspectives dépravées II*], Paris : O. Perrin, Jeu savant, 1955. Éd. Paris: O. Perrin, 1969 : *Anamorphoses ou Magie artificielle des effets merveilleux*. English translation: *Anamorphic Art*, New York : Abrams, 1976.

anamorphosis is a native property of perception in general, in that the human subject will always see him/herself inscribed into the object in the form of a blur or mar. Anamorphosis has to do with the way that appearances are compressed into curtain–like formations seeming to conceal truth behind them.

Thus, one of the first explicit references to projectivity comes in a provocatively Euclidean package: Hans Holbein's double portrait, *The Ambassadors* (Fig. 6). The date of the painting is inscribed on the back, but here begins a mystery that ends with a proclamation of projectivity in all its glory. The date of *vernissage* is indicated as April 11, 1533, 4 p.m. Why the over–precision? Two key facts stem from this clue. First, this date and time were widely held to be, by astrological calculation, the time of the Apocalypse. Part of the rationale involved the over–determination of the number three: 3x500 (historical eras were thought to last 500 years), plus the 3x11 or 33 years of the life of Christ. Even the time related to three. At four in the afternoon, the sun was precisely 27° above the London horizon.

What about origami? To find out this date clue, one would have to turn the painting over. Curators and commentators rarely did this, and when they did, they did not think about the significance of the over–precision.[13] On the painting's *recto*, the number three lies behind the construction of triangles whose sides meet the horizon at 27° (3^3). One of the triangle's sides defines the angle of the famous anamorphic skull that is commonly written off as a *memento mori*, a counterpoint to all of the finery on display, presumably the collection of the two well–known wealthy Frenchmen shown in the portrait. Dolar rightly advises us to move beyond the moment of simple surprise at discovery that the blur can be seen clearly if one is aligned with this 27° blur. Dolar, however, is unaware of the radical geometry that connects the skull–line with the crucifix at the upper left of the painting. Critically, this crucifix is *half–hidden* by a green curtain. It is visible/invisible, just as the numerology of the reticular lines has remained hidden, even to expert examiners, for centuries. The rule seems to be that of Poe's "purloined letter": the more you leave something out in clear view, the more it will remain unnoticed.

Clearly, this is a case where effectiveness (the function of secondary virtuality) is, as an issue, front and center. The Apocalypse will be nothing if not effective. Effectiveness here is partly written in a numerical code (using the number three) and a geometric code involving, appropriately, a triangle. Overdetermination is also the essence of Pappus's Theorem. Two points will determine a one–dimensional line, three points are over–determinative. But, one could as easily say that the two given lines (S2, S2) are over–determined with the third, revealed by "twisting" the points to produce crisscross relations. The third line is the overdetermination, the second virtuality, of the two given lines. Holbein seems to be offering his own version of Pappus's theorem. The planar aspect of the surface of the painting "offer up" a third line, an anamorphic S1,

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[13] The only art historian to notice the date seems, in fact, to have been John North, *The Ambassadors' Secret*.

that engages the viewer physically and relates him/her to the "origami" act of flipping the painting over.

In projective geometry in general, over–determination plays a key role. Because, all lines intersect in projective geometry, including parallel ones, these lines can be re-drawn as passing through a common origin.[14] This is equivalent to saying that "everything is related to everything else." The angles of the vectors from a new common origin determine how they will intersect a "projective plane," and the points of intersection can be scaled, allowing the projective lines to be described algebraically. All this may seem to be too much to take in for a non-mathematician (such as myself). The element of over–determination, however, lets us connect to other aspects of the uncanny, of anamorphosis, of architecture's key functions of void and liminal passage, of the psychoanalytical subject's drives and compulsions, the enthymeme, etc. — all cases where projective geometry structures a "secondary virtuality of effectiveness." These help situation the purely mathematical considerations (folded spaces, double infinities, self–intersecting surfaces, scalers, etc.), not to domesticate them but, rather, to show off the common dependency on the splitting that takes place within pictorial virtuality in all its forms. Euclid covers up this, but projective geometry shows that it is both indispensable and comprehendible.

Holbein's painting was revolutionary, not just for its hidden codes and use of anamorphosis, but because the painting was an instrument of a broad intellectual understanding of cosmic time and the meaning of the Apocalypse. These deeper meanings must have been in place for Holbein to reference them so confidently in this overdetermined "text" about the end of the world. Yet, for all of its projectivity potential (it could easily be regarded as a handbook on projective geometry's more occult relationships), it occurred a century before Desargues' more sober elaborations of projective geometry. Compare this to a much later case of art referencing projectivity, Picasso's *Les Demoiselles d'Avignon* (Fig. 7)*.* There is also a curtain, a

Figure 7. Pablo Picasso, *Les Demoiselles d'Avignon*, 1907. This painting comes close to duplicating the logic of Magritte's cylindrical internal sky. Few commentators have remarked or even seemed to have noticed the sky painted on the inside of the curtain at the back, and presumably also on the curtain at the front. Possibly the bowl of fruit is a reference to the famous anecdote about Zeuxis and Parrhasius, but the seated figure is surely a reference to Melancholy, and Dürer's *Melencolia§1* in particular, considering the anagram, *limen coeli.*

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[14] This is equivalent to saying the the elliptical horizon is equivalent to a point where positive and negative infinities merge! As strange as this seems, it corresponds *precisely* to the (Lacanian) phenomenon of extimity, where the radical exterior is convertible to a radical interior. This justifies the terminology of "super–sphere" to characterize externality (justified by ethnography's many but congruent depictions of Elysium) and the arguments, by Vico and others, about the *conatus* or existential negation of a point associated with divine momentum (God's stillness; Plato's "moving likeness of eternity" in *Theatetus*).

sky painted on the curtain interior, and, in previous sketches Picasso made as studies, a skull marking the entry. A figure with Melancholy's traditional stance figures in the foreground. If Picasso seems late, Melancholy pitches the ball back to Holbein's times, namely Anton Dürer's complex and heavily encoded engraving, Melencolia§1 (1514), a misspelling that is an anagram for *limen coeli,* the boundary of heaven. The engraving, like *The Ambassadors*, is larded with numerical and geometrical references. Clearly, Picasso had some feeling for what Dürer was trying to say about representation as such, even if he missed some of these clues.

Continuing with his collaboration with Georges Braque, and then the work of other cubists, surrealists, vorticists, and others concerned with non-Euclidean, non-linear geometry. In literature, the champion of projective geometry was an unconscious transmitter, James Joyce, who worked out the same curtain problem from a different angle. But, other writers had foreshadowed Joyce's breakthrough in *Finnegans Wake*: Góngora, Cervantes, Shakespeare (cf. *Love's Labour's Lost*), later Cortázar, Georges Perec, the Oulipo Group …. The point is that a substantial number of revolutionary developments continued to run alongside the mathematicians who knew the value of Desargues' work, but Desargues himself was rarely given credit for first figuring out the secret of a secondary ("projective" virtuality lying within or alongside the first, "Euclidean" virtuality.

Euclid is good at locales, saying how things are can be measured accurately for building or shooting at, how land can be cut up and sold off, how forts and houses can be made to stand up. It starts to fail with star-gazing and sailing ships on the open seas. This requires spherical geometry, the favorite study of Ptolomy. What's flat on a sphere is not what's flat for Euclid, and sailing requires a good way to tell time. As for the stars, well, the Assyrians had a better handle on that than Euclid, and kept their (astrology = astronomy) secrets to themselves.

But, what architectural phenomenology says about Euclid is expressly false on several counts. It is not historically, culturally, or mathematically primary. The astrological–astronomical sciences of the Chaldeans, Mayans, and Assyrians was much more sophisticated; and the Chinese were working on it at the same time. Also, as the 19c. geometers who revived the writings of Desargues discovered, projective geometry is foundational. You can derive Euclid from it but not the other way around. Projective geometry lies at the basis of *all geometry*; it is not a quirky offshoot or radical overthrow of Euclidean naturalism. But, the Greeks feared infinity and tried to avoid it at any cost. Projective geometry, in contrast, brought infinity within reach, made it something that could be described, scaled, and (of course) projected.

Architectural phenomenologists seem to think Desargues' bad timing discredits his theory, but — as far as I can tell — they don't try to understand anything about projective geometry. They nonetheless credit this unfortunate genius with a cultural revolutionary amounting to the birth of Modernity. One 1983 text claims that Desargues' *manure universelle* was the first step of a functionalization of reality that would lead to the Industrial Revolution and the crisis of Eu-

ropean science during the 19c. — Really? The scholarship of anti–projectivity authors has rarely if ever been questioned.

In other words, architectural phenomenologists agree with Damisch and Michel Serres that Desargues is pivotal, but portray him as an evil genius. Other historians, including mathematicians, regard him with the highest admiration, putting him before even Euclid in importance. I find it hard to understand why architecture theory isolated itself in this way. The result has been that architecture education in general has been dissuaded from understanding the geometry of "secondary spaces" that penetrate and ground the "primary perspectivalism" of Euclidean pictorialism. One result has been the (needless, in my view) clash with the technologies of virtual reality, the animated 2-d and 3-d computer generated imagery that has been used extensively to "show what buildings look like" (*sic.*) but also to smooth the process linking design with fabrication and assembly. The obvious utility of this last advantage of calculating the architectural fabric with the precision of airplane manufacture has been more than sufficient to dissuade architecture schools from throwing away their computers at the advice of phenomenologists who claim them to be the work of the devil. But, in general schools have suffered by becoming the un-funded training camps for corporate practices that, after graduation, will continue to underpay interns and new employees.

Although much thoughtlessness has been imposed by the confusion of parametric design with projective geometry, *there is no relationship whatsoever.* Parametrics remains entirely within the domain of descriptive and Eudlidean geometry. Desargues is not Descartes. If anything, it shows that the Greeks' desire to avoid the issue of infinity at all costs is still with us. Pascal, Desargues' enthusiastic young collaborator, used projective geometry to define God as an infinite sphere with center everywhere and circumference nowhere, but architectural phenomenologists say that projectivity is about mechanism. This makes no sense.

The uncanny

Projective geometry can be said to be uncanny, but the more important point is that the uncanny, *if we can manage to define it with proper precision*, is also — and equally in its own right — projective. The uncanny does not mean something that is simply bizarre. Its ethnological as well as etymological origins come from "home" (*die Unheimlich*), beginning with a rather contronymic matrix of meanings characteristic of a projective structure: homey and un-homely rotate in an orbit detached from the ordinary as antipodal consequences of the *act of concealment*.

The uncanny's origins are explicitly architectural. The failure of the home's ability to contain and insulate that comes to light.[15] Its sheltering function gives way. Clearly, what remains is of the order of a "pure architecture," an architecture without building (or, with a dysfunction of building) in the same way Deleuze put forward the provocative idea of a "body without organs." "That which ought to be concealed has come to light," notes Freud in his famous study of the uncanny, and in this phrase itself is concealed a wealth of ethnographical practices, rituals, and stories.[16] From "The Fall of the House of Usher" to *The Haunting of Hill House* or even *The Castle of Otranto*, there is an unlimited supply of examples of architecture's engagement with the uncanny, to the extent that one could say that a house that cannot be haunted can never be a home. We cannot tell architecture's story without engaging the uncanny, and *vice versa*: the uncanny is about architecture.

At first, this seems untrue. Ernst Jentsch defined the uncanny's two "atoms" as conditions that were really two sides of the same coin.[17] On one side life: the living person who, fearing death, runs precisely towards death; and, the dead person who has not noticed or has forgotten that he has died. In the first case death is self–constructing and directional. Any attempt to avoid it directly contributes to the fate of meeting it. Clearly, this is a geometry problem. It follows from the general pattern of *Nachträglichkeit* given in Fig. 3. There is an "Euclidean" local linearity that reaches a point at the future anterior, from which springs a "gravity's rainbow" compulsively returning to the origin point, the trauma that had gone unnoticed at the time. The second case, of the dead person forgetting how to die, seems also to be geometric and consistent with Figure 3, if only on account of the wealth of ethnographical data that depict the *post mortem* wandering of the dead as a labyrinth of self–intersecting paths. The labyrinth meander is a fractal series of spaces folded over on each other. Figure 3 is the logical kernel of this folding.

But, the clearest connection between the uncanny and projectivity is that the two conditions, being alive yet constructing death unconsciously (A_D) and not noticing / remembering that one

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[15] This could be seen as a reaction to the attempt, in the name of shelter, to seal off an interior from exterior space. This suggests that "the outside" was not originally perceived as such; that its different features and charged topology constituted a force field blending anxiety and desire, flight and pursuit, safety and danger. Marking off segments of this charged space would require equipping its doors and windows with compensatory gestures of magic and homeopathy to compensate for a theft from the whole. Here, the relation of privation to prohibition is obvious. Cutting off a segment of the whole requires payment that *should be offered* to avoid retribution. This belief and the compensatory behaviors following from it are sustainable only if the whole of space is finite, i. e. circular / curved. Every house is in this sense "the first house." The hearth of the house is, in effect, made to be that part of the circle that intersects itself, the (projective) figure known as the "interior–8."

[16] Sigmund Freud, Hugh Haughton, and David McLintock, *The Uncanny* (Brantford, ON: W. Ross MacDonald School Resource Services Library, 2013).

[17] Ernst Jentsch, "Zur Psychologie des Unheimlichen." *Psychiatrisch-Neurologische Wochenschrift* 8, 22 (August 26, 1906): 195–98; and 8, 23 (September 1, 1906): 203–05.

has died but retaining an element of life creating a projective momentum (D_A) produces the precise crisscross between A_D and D_A that Pappus used to reveal the eigenvector[18] that "fated" the position of two lines by means of a third projective line.

The key here is the geometry of crossing "alive" with "dead" in cross-inscription. The living person has a GPS implanted within that, no matter what direction is chosen, the angle of her *askesis* will bend to meet death, defining a perfect arc. (I call this, following Thomas Pynchon's novel title, *Gravity's Rainbow*). The dead person who has forgotten how to die is a famous ethnographical motif. The dead do not die fully at the point of literal death. Rather, they must wander for a period equal to that of mourning undertaken by the family; they also have a GPS implant that is geared to follow a fractal algorithm, folding and twisting space into a labyrinthine meander "whose center is everywhere and circumference nowhere." Here, too, gravity has its rainbow — an invisible gravity, a vanishing point placed on the horizon lying at infinity but made available to ordinary experience (in that anxiety about death defines life and denial of death defines death). The point of projective geometry is truer to these conditions, as uncanny as they may be, than Euclid, which forbids such infinities, even when localized by cross–inscription.

If Euclid fails to recognize what is essential to human ethnography as well as geometry's own mathematical grounding in projectivity; *and*, if the love of Euclidean locality by architecture's brand of phenomenology only pushes it toward head–on clashes with the rather boring exercises of virtual reality, why is there such resistance to Desargues' admittedly profound discovery? Isn't projectivity actually the essential means of achieving phenomenology's stated goal, of understanding the logic of the lifeworld? It is impossible to think of, or even describe accurately, a lifeworld that is somehow missing the uncanny. Just as impossible (as is evident mathematically) is the claim that Euclidean space is self–sufficient and complete without the more primary operations of projective geometry.

Perhaps the reader feels unqualified to respond to these issues without knowing more about what projective geometry actually is. One remedy would be to study some history of mathematics textbooks, or sample lessons easily found on YouTube.[19] Another might be to look more deeply into the actual, deep intellectual concerns of the artists who took non-Euclidean geometry seriously. These show that the basis of modernism took its cues from what it found in ethnography (masks, dances, rituals, etc.) with a rather commendable sense of responsibility; that the approximations of Perec's lipograms or Roussel's *procédé* were not so far off the mark. Reading Italo Calvino, for example, uncovers a rather specific mathematics of arches, zigzags,

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[18] Louis Kauffman defines an eigenvector as a line that, by not changing, allows all other lines to change.

[19] Norman Wildberger's series on the history of mathematics is particularly useful. See also *Numberphile* podcasts concerning origami proofs, Pascal, projectivity, the Fibonacci series, and many other topics.

cross-inscriptions, and combinatorial palindromes. And, Edgar Allan Poe has some famous ciphers and folded spaces.

Because art and literature connect to projectivity practices in folk practices, they offer the non-mathematician an alternative: *cultural accounts* of what projectivity is, and how it works. The ultimate geometer of projectivity is Giambattista Vico, who in his *New Science*, provides an economical model of projective space required by the construction of metaphor in myth. Architecture phenomenology has praised this reference to metaphor without understanding how it works. Vico said it took him a good twenty years to figure it out; the phenomenologists seem not to spend more than five minutes to think it is the same kind of metaphor studied by hermeneutics, but profoundly it is not. Vico makes this clear. At the origin point of the human subject proper, there is a relationship between the use of signs and the emergence of language proper — two radically different things. This emergence comes about from an excess of the contents of a frame which provoked a contraction of meaning, not a substitution, the customary definition. Excess and contraction make it impossible to say that mythic thought is something exemplary, something "close to nature" that moderns can willfully embrace out of admiration. Vico never says anything like this, but he is intentionally mis-quoted as a hero of phenomenology, which is again a brutal appropriation that entirely misunderstands Vico's theories. Vico is not the hero of architecture phenomenology; he is the enemy. *The New Science* is not hermeneutics; it contradicts Husserl's idea of science, and the "crisis" that Husserl bemoans is a terminus that Vico would not only predict but celebrate. If anything, Vico anticipates the mania of Hegel and hammer–smashes of Nietzsche.

Vico is the poet of the uncanny, not just in history but in the thinking we must do about history, which must drink a bit of the wine it's bottling. Just as Merleau-Ponty drifted toward the idea of "wild thought" in the second phase of his work (*Visible and Invisible*), Vico integrated wildness into the heart of his masterwork, in the form of a rhetoric that can only be called a narrative form of projective, or "secondary" virtuality. To describe this, one must understand metalepsis, the logic at the basis of the joke and the dream. Metalepsis refers to the frame as something with a concealed intention to change the rules of the game it has set up for others to play. It is obvious in jokes following the form of the German–Jewish *Witz*: the two rabbis who meet at the train station, and one says "How is it that you tell me you're going to Krakow, when I know very well you *are* going to Krakow?" Or, the wife at her husband's death–bed who asks, "Darling, I'm so distressed that you're leaving me, what are your last wishes" "—Marry Rosenfeld." "—But, Rosenfeld was your worst enemy!" "—That's right." Or (my favorite but also most people don't get it), "Three elephants walk into a bar and one says, 'Hey, this is a *really big bar*!"

Metalepsis refers to the beginnings, a starting condition, which has too–quickly been forgotten as the action or thought has progressed. Suddenly calling attention to this forces a retroactive realization, that the initial condition required a *double structure*, a kind of mutually accepted

duplicity. This is integral to language itself, where the beginning of the sentence is accepted as "known" although it will be revised only when the sentence reaches the end–point. Then, the meaning of the first words must be revised retroactively. Their initial job of framing the contents to come must be exposed, as having a double and often tricky agenda. When the Cretan Liar speaks his famous lines, "All Cretans are liars," we see him not just a trickster but as someone trying to teach us about reality, the reality of all speaking, that speaking is both an attempt to contain something (a frame) and an act (the making of the frame). This is Vico's contrast between the true, the *verum*, and the made, *factum*.[20] Phenomenologists, and even some Vichians who should know better, have claimed that the *factum* gives us unrestricted access to the true. This is not what Vico said. In his book *On the Ancient Wisdom of the Italians*, he anticipates Freud's interest in the primacy of contronyms, words that mean opposite things, or pairs of opposites that mean the same thing. What is the made, the *factum*? It is this palindromic structure of retroactive meaning, one that surprises us, in that we realize that, although we have made it ourselves, it is something surprising, alien, confounding. Phenomenology not only gets the meaning wrong, but cites *The New Science* as the origin. (*Verum ipsum factum* does not appear once in *The New Science*).

To connect Vico with the hermeneutics of Dilthey, Gadamer (who consistently misrepresents Vico to his readers), and Ricoeur is to miss the real connection: to Jacques Lacan. In his attempt to restore Freud to his rightful position in psychoanalysis, Lacan realized the central value of two key ideas: the "retroaction" of compulsive disorders, that took their logic from language's own *Nachträglichkeit*; and the death drive, the unfortunate name for the subject–as–organism's no–holds–barred struggle to survive, which also involved a backwards cyclic return motion. Lacan, like Vico, realized the centrality of metaphor. What is more amazing is that Lacan, without knowing about Vico's theory of mythic metaphor, came up with a mathematical–looking definition ("*matheme*") that involved the same projectivity used by a "secondary virtuality" that Vico had defined using the metaphor of Jove as god of sky and thunder.

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[20] My advice will not sit well with architecture theorists generally. It is to see Vico's *verum ipsum factum* ("The true is convertible with the made" — *not* "we may know that which we make," as phenomenologists contend) in Lacanian terms. This involves Lacan's idea of the "truth *of* the truth," a recognition that there are not "truths" that do not require a supplement supplied by projectivity, which constitutes an Imaginary where completion must be seen in terms of "local infinities." Vico's *factum* in Lacan's terms becomes the structure of signifying chains that, perversely, yield up the Real because of their gaps, flaws, and failures. At the architectural level, this is the issue of "diarronics," the leakage across lines of opposition between terms that, such as life and death, require uncanny crisscross co-inscriptions. The *factum* is that which is uniquely made by speaking humans. Human language is not the same as the extensive use of signs by other species. As the anecdote of Helen Keller's famous realization of how her use of signs actually constituted a language demonstrates, the act and content of language does not appear as a paradoxical combination for other species, but once it does, language and diarronics are fused.

This coincidence has gone unnoticed, almost entirely, because for various reasons there are few if any Lacanians interested in Vico and few if any Vichians interested in Lacan. Even if there happened to be a small group of unitarians who realized the connections, it would be even more unlikely that they would also think that projective geometry, and its ethnographical, uncanny counterparts, secondary virtuality, the uncanny, and anamorphosis, would hold the key, or at least that the key would allow theory to shoot past the specifics of Lacan and Vico to create an even more diverse theory of the human subject.