

The Double Life of the Uncanny

Don Kunze¹

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Freud wrote in his notes:
“Psyche is extended; knows
nothing of it.”

[Psyche ist ausgedehnt; weiß nichts davon.]



In research as well as personal life, it is always advisable to go to what are called “the primary sources,” but what these are depends on what is secondary. In the case of the uncanny, the primary is not any book but, rather, the practices that give evidence of what we call the uncanny. But, how did we *come to call* these behaviors uncanny, and how did the words we use to frame and describe, and theorize about, the uncanny come into being? These come from books, written by those who thought about the matter and, by publishing, exposed their thoughts to critical review. Our primary source for the ways we have characterized the uncanny, theoretically but also in our cultural conversations, are often texts.

We have a case of someone respecting the idea of the primary source in Freud’s essay on the uncanny, which has been published in several English translations from the original German (“Das Unheimliche,” *Imago* 5 (1919):297–324. The text is short and readable. In it, Freud cites another scholar, Ernst Jentsch, whose work Freud himself regarded as a “primary source,” and even Jentsch’s two articles involve something primary.² Jentsch contends that the uncanny is a *consistent and symmetrical* phenomenon, structured by a double relation. There are two “poles” of the uncanny. In the first, the dead person has not noticed or forgotten that he/she has died. Or — and this is culturally significant — the friends and family of the deceased (1) imagine that their departed is not fully dead; (2) cannot accept the fact of death; and/or (3) imagine that the deceased “is not finished with dying.” Because of the universality of the human respect for the dead, which forbids the exposure or neglect of corpses and in fact requires extensive and elaborate rituals to mark the fact of death, we know this idea must be “hard-wired” into the human cultural psyche. The interval between the first death, which is simply consequential and contingent, and a second death that must meet with certain conditions, is called “between the two deaths.” It is an imagined momentum of life that pushes past the terminal barrier separating life and death. It is the first step in imagining death as something, rather than nothing. In this zone where the dead seem to have some kind of life, the idea of

¹ Following a seminar at the Washington–Alexandria Architecture Center of Virginia Tech. University, Fall 2022 (Camila Mancilla Vera, Marion Eisenman, Paul Emmons, Negar Goljan, and Don Kunze), Mancilla Vera’s research focused on the uncanny; this paper was written in response to that interest and is dedicated to her.

² “Zur Psychologie des Unheimlichen” was published in the *Psychiatrisch-Neurologische Wochenschrift* 8.22 (25 Aug. 1906): 195-98 and 8.23 (1 Sept. 1906): 203-05 (the bibliographical references in the Freud editions do not make it clear that Jentsch’s essay is spread over two separate issues of the weekly).

the soul is born, as *that which is able to traverse the interval between the two deaths*. Because most religions are derived from the idea that there is a soul, different from the body, and that this soul is the basis of the identity of the subject we associate with “conatus” (the preservation of qualities and capabilities *in the face of change and challenges to identity*), we can “reverse engineer” this interval between the two deaths to see that the entire basis of religion comes out of death, the fear of death, and the positive constitution of death, from a negative basis to a positive one.

Jentsch formulated this category of the dead person not fully dead, with a “kernel” of life animating it in the interval between the two deaths (let us abbreviate it as D_A) not as a stand-alone phenomenon. It was the result of a reflection, a mirror of another condition that was structurally similar although its apparent operating principles seemed to be opposite. This was the case of the living person who, wishing to flee from death, runs directly into a trap where death is located precisely in the center.

Jentsch’s other “atom” of the uncanny, A_D , is not simply “the opposite case.” It is as structurally active in the formation of its opposite, D_A , as D_A is structural active in forming it. The two poles, between the two deaths and the living person in flight from death, are co-structuring and co-causal. This must mean, Freud reasons, that it is the distinction between them that is the key to how the two poles work, what their relations must be, and how customs arise that make each pole seem to be independent, distinctive, and supported by separate cultural customs and beliefs. We cannot treat the one without involving the other, or without being responsible for theorizing the cut or distinction that separated them in the first place. The co-determinacy of A_D and D_A is the meaning of these sources of uncanny customs and behaviors, and thus the uncanny itself must be based on the polarity of the uncanny and the cut/distinction that produced this uncanny.

Many commentators on the uncanny, especially the “architectural uncanny,” forget these structural-theoretical obligations and are satisfied to say “what the uncanny looks like.”³ The uncanny does have visible aspects and practices that can be described. But, what about them makes us define and describe them *as* uncanny? Our own theoretical framing practices must be taken into account. How we know and approach the uncanny is itself a part of the uncanny. We cannot claim full objectivity. Modern thought, even disciplined theoretical-critical thought, has something uncanny about it. Thought itself, if it is structurally affected by the polarity of the uncanny and the mysterious cut that separates the uncanny’s two fundamental positions, cannot claim to be not-uncanny. The theoretical catastrophe of self-reference, evident even in the seemingly logical situation of $A=A$, is uncanny under close examination. In one such famous case of close examination, G. W. F. Hegel determined that this supposedly solid basis of logical reasoning was, in effect, good for nothing. Later, almost as famously, Bertrand Russell would present his famous Russell Paradox where Hegel’s argument was re-cast in terms of set theory. Perhaps there is nothing more scandalous than the “uncanny” of the $A=A$, which seems so clear and logical but in fact is easily dismantled.

³ I’m referring of course to Anthony Vidler’s famous book, *The Architectural Uncanny: Essays in the Modern Unhomely* (Cambridge MA: MIT, 1994).

The Story in the Story

We can think further about Jentsch's formula, D_A/A_D . The subscripts behave like the famous literary device known as the "story in the story."⁴ This is the means by which, within a main story, many other stories can be told, usually by one character to others. One classic form of this is the anthology, where a linking tale allows the unfolding of other stories, and even stories *inside* of some of those, to the point where the reader/listener may easily lose track of which story is the "real" one. Of course none of the stories is "real," but the reader has accepted one main line of narrative as a reference point. In a famous case, *The Manuscript Found in Saragossa*, the cover tale is told by an officer in the French army, who found himself under siege at the Spanish town of Saragossa in 1809. In "a small, well-built house" at the remote corner of town, he finds objects left behind by the fleeing resident, but in a corner he notices several handwritten notebooks, written in Spanish, that he manages to save and translate.



Notice that we are already "two stories deep" into the structure of this novel that, when unfolded, will involve what is almost equivalent to a high-rise built downwards into the imaginary space of the story. The novel will be written by a narrator who passes the story over to a character he encounters. That character will, shortly after beginning his or her story, pass it on further to a character in *their* story ... and so on and so on. Even critics who have paid close attention find it difficult to map a hierarchical path connecting the stories and assigning their depth-levels. In some cases, stories-in-stories reference other stories-in-stories, crisscrossing the matrix of stories that any hope for an orderly chronology has to be abandoned.

A similar condition is contrived in a much shorter work by Italo Calvino, *The Castle of Crossed Destinies*. Travels passing through an enchanted forest find themselves at a partly-ruined inn having lost the power of speech. Through gestures, they make it evident to the inn-keepers, a couple whom Calvino describes as, ambiguously, either (1) commoners who, having come into possession of a castle-like building, taken the opportunity to re-invent themselves as a king and queen of a fictional domain, or (2)

⁴ The critical system I will rely on in this essay comes from William Irby's introduction to Jorge Luis Borges, *Labyrinths: Selected Stories and Other Writings*, trans. William Irby (New York: New Directions, 2007). Irby's argument is that Borges used four and only four "devices" of the writing genre of the fantastic: story-in-the-story, contamination of reality by the dream or fiction, travel through time, and the double. The economy of this system is apparent in that each category can be derived from any other, or used as a logical ground of the whole.

an actual king and queen whose domain fell into ruin and its inhabitants had fled.⁵ The hosts somehow understand, and arrange for the mutes to be put up and fed. At the dinner, the guests enjoy an adequate meal but wish to spend longer at the table by exchanging stories. Unable to speak, however, they devise a means of doing this using Tarot cards. An elaborately illustrated deck is produced and the guests draw cards intentionally representative of episodes of their life-stories. The narrator, who is one of the guests, describes each card with the help of expressions on the faces of the story-teller and the gestures used in placing the cards on the table. As the card supply grows smaller, the guests must use cards already placed in the service of previous stories, so the table becomes a matrix of cards. The end result, claims the narrator, is that the matrix represents all the stories that have, can be, and will be told, using rows, columns, and diagonals of card combinations; sometimes only a few cards will be used, in other cases whole rows, columns or diagonals.

Calvino, a member of the famous Bourbaki group of mathematicians and writers who argued for structural necessities in fiction, believed in the principle of *ex falso quodlibet sequitur* (EFQ). This, in short, is the idea that, if a premise is false, all of the subsequent arguments can be true. In fiction, the premise is intentionally false: "Once upon a time," or *C'era una volta*. Every language has a famous introduction used in children's fairy tales, to indicate that what follows is false but may be "true in its own way." EFQ is also a theological principle, the reverse of Dostoevsky's famous contention that "if God is dead then everything is allowed." In fact, history proves that those who believe in God use religion to justify anything and everything. The idea that God is *ex falso* provides the atheist with a way of proving God's non-existence by proving His Effectiveness. The belief in God leads to the idea that everything can be justified as long as it may be said that "God has willed it." Ironically, this proof of God is an Escher sentence.⁶ It proves both the existence and non-existence of God. This "Escher Proof" outdoes St. Anselm's famous proof of God, involving the ambiguity of "that for which there can be nothing greater," which Kant admired because of its involvement of negation.

The Contamination of Reality by the Dream or Fiction / Hamiltonian Time

The formula of "once upon a time" should be realized for what it is: a spiritual insulation. By "spiritual" we mean that something immaterial has been refused passage while something physical has been allowed past the same barrier. We know this through the failure which results in the opposite case: a door is closed but a

⁵ The curious binary nature of the inn-keeper couple could be described as an "Escher formation," a semantic formation that indicates, simultaneously, two opposite meanings or directions, in the same way the artist Maurits Cornelis Escher drew staircases that could be seen as going up and/or down. The uncanny itself is an Escher construct, leading from life to death and/or death to life. Calvino, aware of this ambiguity, provides the reader of his novel with a graphic version of the way narratives might be regarded as Escher constructs. The Escher construct is equivalent to Freud's famous "contronym," the single word with two opposite meanings (1910) "The Antithetical Meaning of Primal Words," *The Standard Edition of the Complete Psychological Works of Sigmund Freud* 11:153-162.

⁶ The cryptic key to this bizarre contention has to do with what the "*ex falso*" of the EFQ claim really means. It may mean, simply, substitution or, more interestingly, suppression (usually by a substitution or marker), condensation (cf. the metaphoric aspect of dream conversion), or the full process of metaphor by which suppression produces, simultaneously, the production of a metonymic chain of signifiers, each of which is anchored to a latent "x" element that is the glue maintaining law and order in the collection of chains and the factor allowing the chains to be mapped onto other media, such as travel across the landscape (as in the case of Freud's famous example of parapraxis, the forgetting of a proper name. Sigmund Freud, "The Forgetting of Proper Names," Chapter 1 in *The Psychopathology of Everyday Life: Forgetting, Slips of the Tongue, Bungled Actions, Superstitions and Errors*. The Standard Edition of the Complete Psychological Works of Sigmund Freud 6 (1901): vii-296

ghost passes through it. The result is that the house is haunted. Normally, and more universally, some object is placed at a door to keep spirits out while ordinary mortal bodies are allowed to pass. Let's call this what it is: idempotency. This is an electrical engineering term designating a switch that, once activated, maintains the activated state. An elevator button, once pushed, remains "on." The elevator will not come any faster if the impatient finger presses it again and again. Only the arrival of the elevator will reset the circuit.

Idempotency, faced with two things⁷, allows for the passage of one thing while restricting the other thing. But, in addition to separating an energy flow into two parts, a side-effect is that the idea of a division has been made possible. The original unified energy, which with the other kind of off/on switch would alternate between positive and negative states, the idempotency switch allows for one flow while restricting another (successive pushes of the button). An example of an idempotency switch is the *mezuzah* (מְזוּזָה), which must be touched on entry to allow the physical person to enter without letting the unwanted would-be spiritual invader. In many parts of the Mediterranean, Near-East, and Asia, custom requires such *apotropes* for all doors and windows.⁸

Idempotency is a kind of Escher sentence relying on opposition logic to create an insulating but selective boundary. If we turn to the case of the story-in-the-story, we realize that just such a boundary is created between every "linking tale" and the subordinate tales in the anthology collection. "Once upon a time" restricts belief, asking it to be momentarily suspended, but it does not restrict all belief. A minimal amount of credibility must pass. There must be enough energy to make characters in the subordinate story on the other side of the "once upon a time" barrier believable and worth our investment of interest. We invest by thinking that a character *should do something or other*, or should be *capable of doing something or other*. No character exists without expectations projected by the reader/listener, expectations that, if proven wrong, will lead to disappointment or even anxiety or, if proven right, will confirm the reader/listener's positive investment. If investment is, as it should be, considered as a form of energy, then we must consider another useful term: the *Hamiltonian*. In any energy system, there will be a major, prominent exchange system, but beyond this there will be neutral or antagonistic forces that are not only significant but critical. In the body, blood flows not simply to carry nutrients and oxygen in one direction and waste products in the other, but also to relay messages from one organ to another. Blood pressure measures the physical flow but not the semantic one. A Hamiltonian account would require including the semantic along with the "liquid" functions of conveyance. The latter could be called "Newtonian" and the former, semantic functions, "quantum." The quantum is secondary but logically primary.

A story system is Hamiltonian. It includes *all* of the energetics that, in latent or active form, allow for narrative procedures and effects. The four techniques of fantasy in fiction, two of which are the story-in-the-story and the contamination of reality by the story, constitute a critical system designed to

⁷ In the case of the elevator button, the two things are intention (to call for the elevator) and desire for the elevator to come as quickly as possible. The idempotent button, by turning on and remaining on, responds to the notification of intention but not the desire. To prove this point, imagine a button that also responded to desire, accelerating the car the more the button was pushed.

⁸ Idempotency algorithms are used by website managers to defend against "denial of service" attacks — floods of requests that overwhelm web site resources, coordinated by automated "bots" — by converting the form of the leading edge of the attack into an "immune response" deflecting later attacks. Medical immunization in general could be considered as a similar means of converting the attacker into a defender, by using a weakened form of a virus to create antibodies able to resist future biological/viral agents. To understand idempotency requires a generic version of the uncanny algorithm, say X_Y/Y_X .

acknowledge and define the Hamiltonian. The immediate implication of this is clear: any truly adequate and useful critical system addresses both latent and active forces operating in a work of art. The inclusion of latency as not simply a remainder or inconsequential element but as a structural principle requires a Hamiltonian methodology, one that allows latency its constitutive and structuring capabilities.

What would make the story-in-the-story and contamination of reality themes Hamiltonian? There are two techniques that have yet to be added: (1) time travel and (2) the double. Both are “quintessentially” themes associated with the uncanny. The arrow of time, so firmly embedded in our expectations of reality as to be unsettling in any failure (such as *déjà vu* or clairvoyance), is grounded in the order of cause and effect. The latter must *necessarily* follow the former. After must *necessarily* follow now and the past that happened before the now. We cannot imagine even minor transgressions of this order without engaging in the uncanny.

Yet, in language, we have continual, ubiquitous, and even formational centralization of this uncanny transgression. The first words of a sentence are not fully known until the end of the sentence, the unit of meaning. The necessary temporal unfolding of speech does not allow the sentence to be completed until the first is revisited and revised by the last. We allow our expectations, based on context and habit, to “hold open” the space created by the first words of a sentence, expecting this debt to be paid off by the ending, but we know full well how easy it is to be surprised. We *suppress* or rather *suspend* our doubt until the payoff is given *retroactively*, in a temporal transaction that seems to lift up the arrow of time in order to slide alternative causalities beneath it. Clearly, this is a form of “a-temporality” that cannot be denied. Despite our allegiance to the logic of time, we repudiate it with every thought, every utterance. We do not know where we start until we finish; but even then, in a fractal-like manner, we insert a new effect-turned-cause into the final moment thought to complete meaning.

Our everyday — every *moment* — use of language contradicts our firm logical belief that time can flow in only one direction. To read this sentence proves the point. In effect, we are “walking Escher sentences,” who contradict ourselves even when we make the claim, by enunciating the sentence, “Time flows ... in only one direction.” Even in this claim, time has flowed in at least two directions. Time is Hamiltonian, i. e., it has the shape and logic of the uncanny. Time “itself” must thus have ...

A Double

The central operator of the equation Jentsch gives us for the uncanny is the “/” between the D_A/A_D . This is a shorthand indicator that there is a pivot, a hinge. One term rotates, spins, flips, or otherwise converts to the other and, in the process, reverses both of its terms. This is analogous to the point at which the Möbius band is created out of a flat strip of paper. An arrow drawn at the ends, consistent with the band’s orientation, \uparrow , is drawn, but when the ends are twisted and joined, the result is $\uparrow\downarrow$, a condition of non-orientation. Topologically, a three dimensional material, the strip of paper, has been converted into a 2-dimensional form, which we can test by drawing a line continuously along the surface without lifting the point to prove that there is only one side. However, this proof is disconcerting. We *see* two sides. And, when we pinch the edges and slide the strip between them, we *feel* that there are definitely two edges, although at the 360° completion of the circuit, we force ourselves to admit that there is really only one edge. Something strange has happened to the idea of the circuit. These proofs have involved the

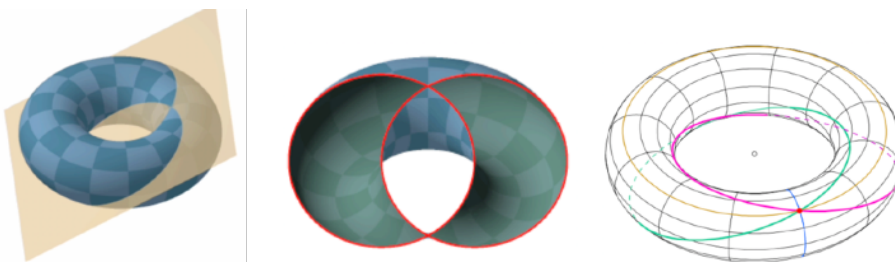
completion of a circuit, but from a perspectival point of view we have traveled 360°, but from a non-perspectival point of view we have gone 720°, or twice around. We cannot resolve these differences easily.

Clearly there is a difference between the “reality” of our looking at the Möbius band and seeing it clearly, with its distinctive half-twist, and the “reality” of our tests using a pencil and two fingers. In 2-d terms, the band is twice as long as it looks to us as we stand in our Euclidean world. Non-orientation combines with self-intersection of two kinds. First, the topological 2-d band is closed, joined. This is consequential to its being self-joined with a twist, or non-orienting. We can see the twist, but an ant crawling along its surface would not see a twist. What is a trap to us in 3-d is a surface without barriers at 2-d. The Möbius band leads a double life, each life however is “dead” to the other, yet each “haunts” the other.

This is more difficult to imagine for a torus, which appears in perspectival space in everyday forms: bicycle tires, bagels, doughnuts, wedding rings. Since all of these forms have thickness, we cannot imagine that they have any relation to 2-d projectivity. Thus, it is important to note that the projective torus is only the *surface* that combines incontinence (the hole in the doughnut) with continence (the air that can be pumped into the bicycle tire). The rule is this: “If you can see it, it’s not a torus.” The torus of these torus-shaped objects lends its surface, and the continence of this surface allows itself to be inflated, filled, cut into, and all the other things that can happen with 3-d figures.

The question is, what makes the torus a 2-d topology? It must have the properties of being self-intersecting and non-oriented. For a point may placed anywhere on a torus, there are three kinds of circles that can be drawn. The first is around the diameter of the “bicycle tire tube,” so to speak. The second is around the diameter of the “wheel” of the tube. Neither of these circles can be reduced. This demonstrates

that the torus’s two voids distinguish it from the sphere, which is an oriented and self-intersecting form. On a trip around the world, the traveler arrives at the point where he/she departed facing in the same direction (orientation), but any



circle on the sphere can shrink to a point. The torus has four circles that resist shrinkage. The two circles that are not the diameters of the tube or tire are called Villarceau circles, after Yvon Villarceau (1813–1883), the French astronomer and mathematician. Each Villarceau circle is made by a *single cut*, but it produces *double, intersecting circles* (the so-named vesica pisces). Without leaving 2-d space, these three types of non-reducing circles specify the presence of two voids, yet there are no barriers or edges with signs that say “do not pass.” One void imprisons, the other doesn’t. This is the virtue of the bicycle tire, bagel, or doughnut. *But...* if we take a knife and cut into the bagel, rotating the knife as it travels 360° around the form, a strange thing happens. The cut produces two mirror-surfaces, each with the form of a Möbius band! *And...* the two surface-bands are linked!

The properties of the 2-d *surface* of the torus as a projective form compel us to admit that it is a projective form if we have to subtract the dimension we use to look at it, in order to consider it as *only a surface*. As a surface, the presence of the two voids reveals a linkage. The generation of the tube-shaped

void is a product of the circular void in the middle of the torus, while the central void is “maintained” by the tube that encircles it. In 2-d terms, the Villarceau circles reveal the torus’s non-orientation and self-intersection by combining them, by being Hamiltonian, an achievement that evaporates as soon as we immerse the 2-d torus into 3-d perspectival space in order to see it with our eyes.

Immersion results in the disappearance of a Real that exists “elsewhere” — in the Hamiltonian relations of 2-d topology. Because self-intersection and non-orientation disappear so completely for the torus, we find it hard to believe in this alternative reality. Why doesn’t the torus show us a twist, like the Möbius band? Or, a cut where we must imagine the inside surface sliding along to the outside as in a cross-cap? Or, a glass neck that penetrates the “bottle” of the Klein bottle? We ride our bicycles, eat our bagels and doughnuts, oblivious to the torus’s projective properties of non-orientation and self-intersection.

In the same way, we find that the “/” of Jentsch’s uncanny formula, D_A/A_D , to be inconsequential; nothing more than a flip or crisscross exchange. We forget the complexity of two elements each serving in two roles, both as a dominant event and latent content, each charged with the duty to convert once on the other side of the “/.” We deny its topology, its 2-d-ness, because in its “immersion” into perspectival reality, we see only customs, rituals, door-charms, and stories about traps. However, the D_A/A_D also has its voids; and its voids also interconnect. It has its internal mirror, and a cut that produces two “chiralistic” faces. If we split apart the D_A/A_D as we would cut into an apple, we would see something similar to the two Möbius band-shaped faces we see when we cut into a torus with a rotating knife. We see the double. We must not deny our denials!



Castor and Pollux

The story of Castor and Pollux can be found, well-told, in Robert Graves, “The Rival Twins.”⁹ Born of Leda from two fathers, one mortal, one immortal, the twins retained these qualities of the fathers.¹⁰ Castor was mortal, Pollux immortal. This did not diminish their strong friendship. So, when Castor was killed accidentally, Pollux confronted the gods of the underworld (some say Zeus) and demanded the return of his twin brother. This was granted on the condition that the twins take turns occupying the realms of the living and the dead, implying that the two would never again meet in their cycle of death and rebirth.

This, clearly, is a variation on seasonal myths, where winter and summer are ruled by gods whose rivalry results in battles fought at autumn and spring, with predictable victories after sometimes protracted battles. The twins themselves are models of the D_A/A_D schema. Originally, Castor belongs to death, as a mortal who will die, while Pollux is life immortal. But, the opposite ordering could be argued, since in death there are no changes, everyone is in a permanent state, a kind of negative immortality. In contrast, life is finite and the thought of death’s termination conditions mortality with its dread. Whichever brother is assigned to whichever category, each takes a turn being alive, then dead, in the final solution of circular

⁹ *The Greek Myths* (New York: George Braziller, 1955), 74: 245–251

¹⁰ In Lacanian psychoanalysis, the theme of the two fathers is a critical point. This makes the “name of the father,” the hallmark of the Symbolic and, by extension, the neurotic’s submission to the phallic law, into an Escher sentence. Lacan in fact employs this in his employment of the torus’ “fundamental polygon” diagram, where the repetition of speech is expanded into two contronymic (Escher) positions, one inside the Symbolic, the other outside. See Seminar XIV (1961–1962), *The Logic of Phantasy*.

exchange. The inscribed elements, the small A and D within the major terms, indicate the function of rotation, just as the torus's surface itself specifies a central void, the guiding pivot of its own self-intersection.

Rivalry, the aspect of the double most associated with René Girard's work, has to do with the subjective anxiety, that one can be replaced.¹¹ There is always someone better, someone better looking, someone with more of anything. This is a fear of premature burial: that one will not have died technically but that, symbolically and socially, replacement has amounted to the same thing. This is one of the categories of D_A , "between the two deaths," but travelling from the Symbolic side of this interval, from being excluded from networks of symbolic relations, waiting in a detached way for actual death to happen, like the character in Oliver Hermanus's film, *Living*, Nighy, called Mr. Zombie, modeled after Akiru Kurusawa's Kanji Watanabe in *Ikiru*. The rival algorithm frees realistic literature of the obligation to refer to the uncanny of the underworld. The same effects of Hades are available in plain air, on earth, in familiar life situations. Nonetheless, the topology of the rotating twins dividing their time between zones of life and death is the same. Only small adjustments need be made to make the twins rotation deal acceptable to audiences with variable abilities to accept fantastic elements and themes.

What makes the double the easiest of the four themes of the fantastic to serve as a guide to the others? Is it the reversed temporality by which Castor must circle *back* into life as his twin enters into darkness? Is it the users-guide that twinship provides in constructing the insulations that allow for differential passage of bodies and spirits in the story-in-the-story and travel through time? Isn't contamination a matter of the uncertain quality of the insulating container that allows for certain kinds of leaks and break-throughs? Possibly the strongest argument for featuring the double as the flagship of the four Borgesian themes is a matter of the Hamiltonian principle, the ideal of including latent along with manifest elements of the D_A/A_D schema. If the themes are about the energetics of writing and reading ... or even more ambitiously about thinking and living ... then the "useless" must be retained and preserved alongside of what seems to be most evidently useful. If the inscribed elements are standing for all of the latencies in the binomial A/D , then it has to be noted that they are the basis of the rotation function, the $/$, which makes each face of the AD the mirror of the other. This chiralistic operator is like the void in the torus. At the proper level of the 2-d projective plane, it is invisible; it exists only as a necessity of the form's relation with itself. Of course, we can see the void clearly if the torus is immersed into 3-space. But, at the level of projectivity, we have to appreciate that the incontinent void is present only as a mathematical consequence, a "genetic proclivity" for the tube, whose own void is present because the circle that defines it is forced into an open position, a spiral. This spiral in turn is rounded into an orbit around the middle void, equally invisible at the level of 2-d projectivity.

Two circles, two forces of rotation, one with a gap that specify a void (the void of the tube), another that from its center pulls the tube around to intersect itself. What could this possibly have to do with the double, let alone the story-in-the-story, travel through time, or the contamination of reality by the dream or story? Did Ernst Jentsch, in his wildest dreams, imagine that anyone would be subjecting his seemingly simple formula to such an analysis? Our approach has been to plunge the formula D_A/A_D into literature and art, where there are myriad examples, whose first grouping around the forms of the fantastic constitute

¹¹ Among René Girard's many publications, some have had widespread popularity: *Violence and the Sacred* (1977), "To Double Business Bound": *Essays on Literature, Mimesis, and Anthropology* (1978), *Things Hidden since the Foundation of the World* (1987).

the easiest correlations between the uncanny in its most compact expression and categories that we can recognize through descriptive and utilitarian features of the four themes. We have seen twins in everyday life, we have all experienced *déjà vu* and precognition. There are hardly any narratives that do not contain other narratives, and everyone has had some experience of a life–event seemingly drawn from some fictional situation. We already have a considerable collection, even before we turn to fiction proper for more examples.

Topology figures so strongly in the rotational arrangement of Castor and Pollux, where we are tempted to say that the rotation is a torus rather than a simple circle, where we more quickly find a way to show how the latent elements play a role at the 2-d level although they are seemingly invisible at the 3-d. The twins deal is “circular,” but it is clearly more than a circle. There are two antipodal points, each occupied by a twin. There is a line holding them apart, a line that, in traversing a void, takes the place of the void. There is the energy that rotates this axel and the wheels at its ends, but this energy seems to be a pull rather than a push, since each of the “wheels” constructing the tube of the torus moves forward because it cannot find its starting point and must spiral forward to repeat the search. The wheel has lost itself, its repetition



Chaplin and the policeman running on a revolving platform, whose central cone is all we see in this photo. Chaplin, puzzled by the fact that he, in running away, has ended up running toward the policeman politely greets him by tipping his hat. Slightly earlier in this scene he has checked his watch.

generates the surface that is the only materiality of the torus. The wheel is the motor of the torus, its energy. Yet, in Hamiltonian terms, we must include not just the two wheels, the D of the A and the A of the D, but the _A of the D and the _D of the A, the elements which, if the D and A are wheels, the _A and the _D are the gaps. If all this is true, then the “/” is the axel holding the two wheels (two twins) apart and moving in coordinated opposite directions.

In a remarkably parallel situation, Charlie Chaplin’s *The Clown* (1928) shows how a 3-d perspectival space circle can become a 2-d projective torus. Chaplin is the unemployed bum pursued by a policeman, whom he diverts into the circus tent and onto a revolving platform being used by clowns in their scheduled act. At first the policeman is behind Charlie but as Charlie runs faster he is close to the back of the policeman, so close that he is uncertain what to do. If he slows down, he comes within the policeman’s grasp, if he speeds up, he is nearly in the position of apprehending his “rival.” Here we have a condition of non-orientation and self–intersection. The circle has become the torus!

Do we have the required two voids? Yes. The void that causes the circle around the “continent” void propels the two foes without allowing them contact. This would end the scene, as actually it does when the two fall off the revolving platform. Are there antipodes? Yes. The policeman and Charlie. Do they constitute opposed domains, as do *Alive and Dead* in Jentch’s original formula? Yes. Charlie represents the lawless but all–too–mortal freedom of the un-policed subject, the Policeman, thanks to his uniform, is immortal. It may not be Mulligan or O’Riley in the uniform, but there will always be a uniform. The immortal twin is a universal, like the divine Pollux.

With antipodes, the remaining question concerns the bar, the “/.” Does it maintain distance? We see that this function exists in the ideal shape of the platform, which means to be balance off by two bodies,

but our anxiety over the fact that Charlie, in running *away* from the policeman is actually running towards him (a literal example of A_D). The policeman, as D_A (his uniform keeps him between the two deaths, until his retirement from the force), can only move forward even though he can almost feel Charlie breathing down his neck. To turn around would result in falling off the platform. The void, represented here by a cone that cannot be mounted, represents an internal fall, should anyone try to climb it. The toroidal path is the only one allowed, and this is defined by the lack that propels both of its agents.

The Katabasis

The Circus justifies our temporarily labeling A_D and D_A “the Tramp” and “the Policeman.” As toroidal doubles, they endorse the speculative move back to Jentsch’s atomistic model to say that it is, in essence, the story of the doubles, the twin, moving between the realms (as it says it does!) of the living and the dead. Just as the uncanny led us to the themes of the fantastic, all of which are summed up by the double, the double, in its engagement of projective geometry in the rotational voids of the torus, brings us back to Jentsch’s crisscross binary. Are we to conclude that the entire (Hamiltonian) uncanny is toroidal? This may be because of the way the binary directly incorporates latent elements in a schema that makes one term the mirror of the other. We have seen that the uncanny is more than a characterization of the preoccupations that, in modern terms, seem “weird.” A characterization can be freely applied in any way; there is no system, no possibility of a science, no basis for hypothesis-formation or experiment. The uncanny tempts us into such characterizations, where at most we can identify similar practices and compare cultural variations.

However, if the involvement of projective geometry in Jentsch’s primary formula, already symmetrical, already Hamiltonian, already topological, is more than a characterization, if in fact the torus constitutes a special “logic of simultaneous inclusion and exclusion,” then we may use the torus not so much as a visual or physical analogy but as a foundational guide. The distinction between the topological torus and the visible torus is critical. The former is 2-d, the latter 3-d. The rule is: “If you can see it, it’s not a torus.” The dimension of view (the sagittal), the visual frame, distinction of figure from ground, and other protocols of visibility convert the 2-d form into a 3-dimensional object. However, this does not prevent us from using the torus as a 2-d phenomenon as we inspect it in 3-space. It simply means that we subject 3-space to “toroidal calibrations,” reversing the process by which we use vision to domesticate what is, in the visible world, *primarily uncanny*.

Looking at a torus we do not see, unless we superimpose imaginary lines that have specific projective qualities, two voids, spiral generation, bilateral structure, or Villarceau circles. In the bicycle tire we see spokes and a hub in the place of a void, in the bagel we clearly see dense, cooked bread. We tend to the spokes and hub to make the bicycle wheel function, we check the air pressure that fills the “void” (now occupied with compressed gas) of the tube. In other words, all of the properties that make the torus uncanny are domesticated once the 2-d shape has been *immersed* into 3-space. The same immersion that makes the torus so useful in perspectival space allows us to fabricate the Möbius band by twisting a strip of paper. Perspectivally, the twist is evident and must be subjected to tests (the continuous pencil line, the pinch) to “prove” what is now not obvious, that the Möbius band is “really” a two-dimensional surface that we “happen to be able to see.”

In ordinary visual experience, two dimensions are all that interact directly with the surface of cells that make up our retina. The eyes must learn to see depth, which is constructed through a complex of interactions with other senses, all of which are produced by the interactions of energy with cells arranged as surfaces. There is no “depth sense organ.” Depth sense emerges from a set of sensations and reactions to those sensations. If the realization of 3-space is considered to be a result and not a given, the same for everyone with sight, we should write the order of senses as 1+1 (the stereognostic arrangement of eyes, ears, hand, feet), a gap, then the result, 3-space (depth perception). Inserted into the gap is a “4” that should more properly be written as 2, because the depth dimension involves time, the fourth dimension, but time in its retroactive, “linguistic” function. The fourth dimension in depth perception requires a temporal insertion in the same way the beginning of a sentence requires the terminal conclusion of the sentence to “correct” the beginning, with a retroaction that suspends/suppresses the beginning until the end returns its verdict. The $12\wedge 3$ or $12/4\backslash 3$ model says, that without this temporal insertion, the appreciation of Euclidean space would be impossible. The 4 involves muscular tensioning as well as movement (virtual or actual). The figure must be able to “slide in front of” the ground. The observer must be able to at least imagine walking to different viewing positions or completely around a physical object to confirm that it’s three-dimensional and not a stage prop.

The reality of 3-space’s gap, filled in with time, allows this interval to be defined differently by different cultures, also for variations at all levels within culture and even down to the level of the individual, where mental states — moods, anxieties, desires, etc. — are able to make modifications in the perception of depth. If we give a name to this modifications, it would be parallax, the relation of the figure to the ground that can be affected by either a movement in the object or by the observer. Parallax can be implied, as when we attribute different views to observers standing in different positions. Camera lenses can simulate even the parallax change of a linear approach toward or retreat from an objects, which flattens or thickens the depth of field, respectively. In Alfred Hitchcock’s famous shot of the church bell tower in *Vertigo* (1957), the zoom lens and camera position were set in opposition to each other. As one flattened the field to simulate increased distance, the zoom lens cut in closer to simulate decreased distance. The immediate result was the dizziness of vertigo, which we could, from this technique, now define as “simultaneous running away and falling toward.” This is the logic of A_D , which is often described with the story, “The Appointment in Samarra,” where a servant who sees the devil in the marketplace flees in terror to the safety of Samarra but instead finds the devil waiting for her there.¹²

When there is such close correlation between a filming technique, as literal as the adjustment of a photographic lens and camera tracking position, we can look at other technical set-ups to see how the uncanny is to be found in such physical, measurable examples. There is another physical, measurable example that is by far more common than the tracking–zoom shot: the descent theme (*katabasis*), a narrative device so universal that it deserves to have its name given to the entire operational field of the uncanny. Indeed, it is the official name for that part of the uncanny, the D_A , known as “between the two deaths.” Here, the story follows the exploits of a character who appears to be living but is compelled the visit the zone of death. Does the character know that it is in the zone of death? Does the audience? How is

¹² The original version of this story can be found in the Babylonian Talmud, *Sukkah* 53a.5-6: <https://www.sefaria.org/Sukkah.53a.7?lang=bi>. It was subsequently re-told by Somerset Maugham in his play, *Sheppey* (1933) and his epithet was used in a novel, *Appointment in Samara* by John O’Hara (1934). ‘Appointment in Samarra’ *Wikipedia*. https://en.wikipedia.org/wiki/Appointment_in_Samarra

something given this identity? These ambiguities may be allowed to persist. There is no requirement in fiction for them to be resolved. To take a trivial example, a travel narrative can be a *katabasis* as long as it involves key elements. But, the story of leaving home, finding adventures, and returning home may not once reference the possibility that the traveler might be dead and not know it, or that the goal of his/her travel might constitute a divination. In Apuleius's Late Latin novel, *The Golden Ass*, the hero Lucius accidentally transforms himself into a donkey and must wait for a full year to eat the flower that is the antidote. His experiences are hellish but why do we need to give this literary form the name of *katabasis*?

Lucius's experience as an animal closely involved with everyday life allows him to be the point-of-view character whose parallax is that of a monster, a man-beast. In the disguise of a donkey he is yet able to see and think as a human. It is *what he sees and experiences* that becomes monstrous as a result, and the reader, through the monster's eyes, sees a world of monstrosities. Lucius's donkey-world is grotesque, not just because, superficially, people treat him as an animal but he perceives this treatment as a human, but because only by taking a point of view of an animal can the audience of this book see the world *in its monstrosity*. The agent converts to the act, and the act inverts the figure-ground relation. Lucius is the agent of a toroidal transformation. The book could easily be re-named, *The Toroidal Transformations of Lucius*.

Here we are in the position to do more than paraphrase this famous ancient novel. Instead of re-telling it, instead of saying that what happens to Lucius is uncanny, we can show how Lucius's transformation — *any* transformation that inverts the parallax of the (required) point of view allowing the reader to be transported into the interior of a fictive space — is a matter of being in two spaces, *two parallaxes*, simultaneously. In reading *The Golden Ass*, the reader is performing a *katabasis* no less than the main character, Lucius. The *double* transformation justifies our comparison of this book, where the *katabasis* begins as simply the template for describing Lucius' descent into animality, to a full-blown “double *katabasis*,” where the journey into the hypothetical “underworld” of a man trapped inside the skin of a donkey leads to the ability to see the world as an underworld.



They Live!

A film beloved by the Slovenian critic Slavoj Žižek, *They Live*, focuses on two key elements of this double *katabasis*. The *Wikipedia* entry:

They Live is a 1988 American science fiction action horror film written and directed by John Carpenter, based on the 1963 short story “Eight O’Clock in the Morning” by Ray Nelson. Starring Roddy Piper, Keith David, and Meg Foster, the film follows an unnamed drifter who discovers through special sunglasses that the ruling class are aliens concealing their appearance and manipulating people to consume, breed, and conform to the status quo via subliminal messages in mass media.

This simple-sounding film seems to lack the basic components of the *katabasis*, but in the sense that every story is technically a descent into a fictive zone where tests are applied and the return trip is held in suspense, time is suspended, contamination-control is an issue, and the reader is doubled by the construction of a point-of-view, fiction itself qualifies a story as a descent experience, no matter what the story may be about. The work of art is a *katabasis* in a technically accurate but

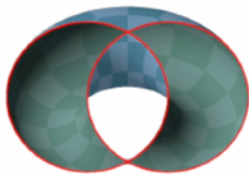


conceptually profound sense, meaning that our entire experience of art could be — should be — considered as an Underground experience. Within this, following the logic of stories–in–stories, we may encounter other descents, in disguise or fragmented but still descents nonetheless.

In *They Live* the magical sunglasses enable the user to “see things as they actually are.” What in fact is this existential truth–state? First, the glasses *subtract* color

and replace this objective dimension with messages that appear in large letters across walls and buildings. The intent is to show that, within ordinary spaces and objects — in fact within the *ordinariness* of these spaces and objects — there are latent messages whose very latency compels us to follow commands of an organized, demonic Other, whose wishes we do not — cannot — fully understand. Even when the message seems positive, as in Lacan’s classic Enjoy!, the result is sinister. “Enjoy what? ... How?” This command means that, even when we think we are doing something that is personal and enjoyable, we are *being enjoyed* by the Other, that our actions have unseen and unintended consequences that form a part of an invisible libidinal economy of the Other. This is the full and ultimate meaning of the Hamiltonian: that to correctly understand the economy, the exchanges of energy that constitute the network within which each object, each action, each agency, each subject participates “locally,” the same objects, actions, agencies and

subjects are nodes in a “global” exchange where the results are harvested according to algorithms that cannot be translated or understood at the level of the component parts.



The (katagraphic) cut through the torus’s two voids is mapped by two Villarceau circles, which form a *vesica piscis*. The central void of the torus forms an edge where the center of each circle is at the periphery of the other, a model of inside-outness (Lacan: *extimité*).

This is a secularized version of the theological view of Providence — the belief that our actions, which seem to be knowable at the level of the individual, play an entirely separate role at a higher level, where the quality and quantity are converted into alien currency. The two schemas, local and global, intersect along the lines of the vesica piscis of the Villarceau Circles. Can the vesica circles be considered *primarily* as toroidal, and not just as a kind of Venn diagram? The key is to distinguish Venn circles from Euler circles, which are what the cuts on the torus made by the Villarceau circles are, technically. What are Euler circles? Euler circles show, in contrast to Venn circles, *only that which can be encountered in everyday life*. This is a mysteriously open-ended and vague definition. However, it can be explained by the unexpected relation of Euler circles to the strange comportment of victims of the condition known as aphasia.

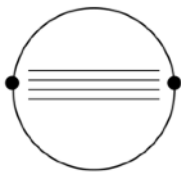
In *Phenomenology of Cognition* (2021: 294), Ernst Cassirer describes the curious response of aphasia victims to say or do something; if asked to do something contrary to the aphasiac’s perception of reality, they cannot bring themselves to do it. If asked to describe something contrary to what they see to be the case, they refuse, without being able to explain why. Cassirer reports his encounter with the aphasiac:

He is unable to *repeat* anything other than the “actual” state of affairs that as such corresponded directly to his own concrete-sensible lived-experiences. In the course of a conversation that took place on a bright summer day, I pronounced the sentence, “It is bad, rainy weather today” and

asked him to repeat this sentence but he was unable to do so. He said the first words easily and surely, but then the patient faltered and paused, and nothing could induce him to complete the sentence in the form giving him; each time he slipped into another form of the sentence that corresponded with the actual facts.

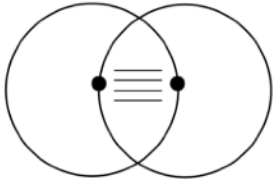
Another patient in Frankfurt, who had lost the use of his right arm, when asked to say “I can write with my right hand” would always replace the word “right” with “left.” When Euler circles combine so that the periphery of each runs through the center of the other, a central void is created. This is not an “empty space” *per se*, but something that resists transgressive filling or assignment. The void is not just a nothing; it is a “nothing of nothing,” a void of a void. This is the kind of truth that the wearer of the spectacles in *They Live* see. Not only do they see messages that have been concealed by the ordinariness of things, they see that ordinariness itself as been the instrument of concealment; that it has conscripted them as willing accomplices to “see what they wish to see” and suppress the latent content. The result is that the blind subject is blind through the very act of seeing what the Other requires it to see, and become involved in (metonymic) signifying chains where locally present meanings are governed from a remote position beneath or behind a bar, a kind of /x factor, a “remote control” in the hands of a capricious Other.

In short, *They Live* is about a “they” that lives through the lives of phenomenal “others” who play a role in a play written, directed, produced, and viewed by alien entities, unknown and unknowable. This mysterious otherness in the film points directly to the lack of the sagittal dimension in projective space. This is the dimension by which we see and subsequently relate the visible world to other experiences. This is the way we collate and compare the “facts” of these experiences into formations of knowledge, and the dimension by which we return from theory to the world to corroborate hypothetical assertions. Without this dimension, there is literally no way to know anything. Within the sagittal-lacking projectivity of the torus, there is nothing, and all the vesica’s void can do is point to a location where there are *two* types of rotation. Like the truth-telling aphasiac, there is no distance that allows theories to acquire the distance required for contingency (conditionality). Without this dimension, theory is pressed flat against the Real, and the Real is indistinguishable from the theorized. The 1:1 flatness of the world revealed in *They Live* is not just an ideological messaging system revealed by peeling back a veil of concealment, it is the exposure, arrest, torture, and execution of the participating subject! It is the collapse of the dimension — the *parallax* — that would allow the subject to “have a relationships” with the world. No sagittal, no parallax, no ability to say what the world is like or how one can live in it.



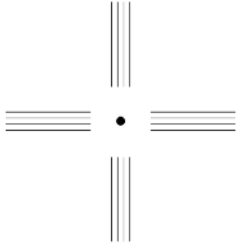
In the projective plane, every line is a member of a family of parallel lines that actually meet at a point, or rather two antipodal points.

This is what happens in the *katabasis*, and it can be drawn: there is a passageway, which can borrow terminology from projective geometry to say that it identifies as a “one-dimensional subspace.” The projective line is a line and a point, or actually two points. The point is a vanishing point, where families of parallel lines *actually* meet, in contrast with having only the appearance of meeting in Euclidean geometry. This actual convergence is duplicated on the other “end” of the line, • — •. The two points lie on another line that is fictional for Euclid but actual within projective space, the circular horizon. This little solar system is fully rotational, which is to say that position doesn’t matter. The horizon equalizes; the only definitive relation is between the family of parallel lines and its two points “at infinity” on the horizon’s circle.



The identification of the vanishing point and viewing point in the projective plane could be called the “vesica” condition.

At the same time, the vanishing point is indistinguishable from the center of this solar system, because about each vanishing point there is no specific direct from which parallels must converge. Any and all line sets that are parallel will pass through this point. Yet, the central point does not forget its antipode, the fact that it is one of a pair of twins. The graphic solution to this seemingly paradoxical twinship is that each point occupies the center of one circle and, at the same time, the periphery (horizon) of another, for whom the relation is obliged to be symmetrical: the vesica pisces. This is the “meaning,” or rather “configuration” of the void of two Euler circles that have overlapped, creating what Lacan called a “union without intersection.” The *katabasis*, which by definition subtracts the sagittal dimension of display, and thus forecloses description, hypothesis, and other parallax conjectures, creates the void that theological/literary tradition has named Hades.



When the vanishing point is equated with the viewing point (they are co-determinative), four families of parallel lines seem to converge and pass through this one point.

How does one “visit” Hades? Normally, the rule is that the dead soul, in absence of its body (= the subtraction of the sagittal and its accompanying parallax), is still able to move, at least in fictional accounts. How does it move? This is both an architectural question and a theological–existential one. Yet, it must be answered according to the rules of projective geometry, where parallax has been suspended and new conditions imposed. The vesica, the profile of Villarceau circles, a.k.a. Euler circles, is one of those new conditions. We can check out this in the real life of our “aphasiac” experience of Chaplin’s *The Circus*, where Chaplin loses orientation (running from becomes running toward) at the moment he nearly touches the policeman, who is “dead without knowing it.” We can even now name this “not knowing it” properly, as *kenosis*, the tradition by which knowledge is acquired by subtracting parallax through meditation, fasting, and prayer (cf. Jesus’s training with the Essenes).¹³

From the traditional literary presentations of the *katabasis*, Book VI of Vergil’s *Aeneid*, for example, we can say that, if the aim of the descent and return of a living hero into and back out of Hades can be said “to learn something,” then the form of that knowledge is *kenosis*: that which I know but don’t know that (or how) that I know. Kenosis is lacking the parallax dimension by which knowledge is compared to “taking views.” It is a scene-less scene, a frame that points back at the viewer at the same time it seems to point in the distance.

As complicated as this may sound, it has occurred to others that they should attempt to model it. In Canto III¹⁴:

¹³ We know the result of this parallax subtraction through the Biblical literature descriptions of Jesus’s responses to requests and questions, where his “koans” take the form of the Jewish *Witz* joke, a style of comedy that turns back to the questioner as the source of the answer: Let him who is without guilt throw the first stone. Even in the seemingly practical advice, “Render unto Cæsar that which is Cæsar’s” are the functions of non-orientation and self-intersection critical to the *Witz*. Certainly there is an element of *ex falso* in all of the metaleptic formulæ of comedy that refer to the frame at the same time content is addressed. *Ceci n’est pas une pipe* is the classic modern reference to this, where the circularity of the Jentschian uncanny is evident.

¹⁴ Quoting from Lacan’s Seminar XIII, *The Object of Psychoanalysis*, January 19, 1966, 88–89.

And I, to admit that I was put right
 And convinced, as the case indeed required,
 Raised my head to address some words to her;
 But an apparition appeared, which held me
 So closely to itself, to look at it,
 That I did not remember my confession.
 As through a glass which is transparent and polished,
 Or through tranquil and translucent water
 Which is not so deep that it is dark at the bottom
 The outlines of our faces are reflected
 So faintly, that a pearl on a white forehead
 Does not come less readily to our pupils;
 So I saw many faces set to speak:
 Which made me run into the opposite error
 To that which made the man in love with the pool.
 The moment I caught sight of them,
 Thinking that they were reflected images
 I turned my eyes to see whose they were;
 And saw nothing and looked back again
 Straight at the light which came from my sweet guide
 Which, as she smiled, blazed from her holy eyes.

In this explicitly geometrical explanation, Dante employs the literal function of the projective line, with the two vanishing points produced by the mirror of Beatrice's eyes. These rays reverse, from being intromissive (passive) to being extromissive (active). At the same time the space between Beatrice and Dante is removed, and the faces Dante sees appear to be *behind* him. Extromission has turned the viewpoint around 180°, but in reality this reverse view is the *reversal itself* of Beatrice's optical line of sight. "The man in love with the pool" is clearly Narcissus. Dante makes the same error as Narcissus, taking the reflection of many faces as those who are standing behind him. Dante sees a "parallax inside parallax," a

kind of stereogram that, thanks to its 2-d topology, is able to present a sense of perspective although it is flat and surrounded by space that seems to have the advantage of the sagittal.



As the haunted mirror takes over the mind of the fiancé, he is able to see the background of the 19c. room in which the former owner of the mirror had spent his last days; his thoughts become those of the jealous husband.

Is there a way to make this dramatic moment in *The Divine Comedy* more understandable? In Arturo Cavalcanti's film *Dead of Night* (1945) a collaboration of directors created an anthology film with five linked stories enclosed within the covering tale of the architect Walter Craig's visit to a farmhouse he is to enlarge and refine. When he arrives to look around, he finds himself in the midst of an ongoing house party, with guests he feels he has met before, in various dreams or perhaps a continuous segmented dream. Craig is astonished by the fact that he knows the stories of the guests in advance, and when they sympathize with his astonishment, each guest in turn tells about his or her own encounter with the uncanny.

One story in particular relates to our current problem of a parallax that is subtracted then added back to a space. A wealthy socialite gifts her fiancé with a mirror she has found in a rural antique shop. Until the sale, the mirror had not been used since it was hung in the bedroom of a wealthy landowner, disabled in a horseback riding accident. Unable to move, he becomes increasingly jealous of his younger wife and imagines that she is enjoying his paralysis with a succession of lovers. Unable to control his passions, he manages to stagger from his bed, seize and strangle her to death. The mirror has “seen” this hideous act and “not forgotten it.” The image lies latent until the socialite picks the mirror out on account of its elaborate Chippendale frame, to give to her fiancé as a wedding present. The fiancé is at first delighted to have this expensive object in his bachelor apartment, but he soon notices something strange about it. Instead of seeing himself in his bedroom, he sees, behind him, the bedroom of the mirror’s original owner. Mirrors are implicitly narcissistic, since we use them to correct our appearance before we present it publicly. But, this narcissism converts to the jealousy of the original impassioned owner of the mirror. The force of that passion turns into a kind of extromission of the mirror — the mirror as *looking at* rather than *being looked into*. The result is that the new owner becomes obsessed with his wife’s imagined romantic affairs, despite the fact that they are together nearly all of the time, in the bliss reserved for newly-weds.

The situation begins with only periodic replacement of the ordinary ground with the remembered one, the murderer’s bedchamber. As it intensifies, the mood the fiancé sours, and he thinks that he must kill his wife. At a critical climax, he attempts this but the wife manages to smash the mirror and break the spell. Although this is little more than a fairy tale for adults, it has all of the features we need to show the relationship of the 2-d projective plane, the space where parallax is forbidden, and the *katabasis*, where in this film is the descent into madness. The story is both uncanny — strange, inexplicable in logical terms — but *structurally* uncanny in the way that a space, a void, is created *inside* a 3-d Euclidean reality.

Also, here we have the themes of the fantastic. There is travel in time, as an event in the 18c. is preserved, transmitted, and re-effectuated in the 20c. There is also the fact that this is a story situated inside another story, alongside the others the guests tell about their own encounters with *die Unheimlich*. The contamination theme is clear: the mirror’s representation of reality becomes a reality that becomes superior to the dominant reality whose job it was to reflect. This reversal of the role of the (passive) reflection is also a reversal of effect for cause. Finally, there is the obvious creation of the double: the previous owner and the present owner of the mirror.

With the four theses of fantastic literature so compactly presented, we are able to use this short scene to untangle the projectivity relations of parallax that are the final step in associating the uncanny with the “one dimensional subspace” of the underworld, the ethnographical account of the *katabasis* that proves the (toroidal) topology of the uncanny. We will, in the progress of this “proof,” find that the role of Villarceau circles in equalizing the roles of the vanishing point and viewing point — the double negation encountered in *They Live* — opens up a vast territory of examples that might serve as living laboratories for how projective geometry interacts with perspectival geometry. The “trivial” example of the stereogram, the illusion of a 3-d space produced by the overlapping pattern’s small differences, presents us with an ultimatum: that parallax is created *neurally*, “in the mind” so to speak. This happens on the occasion of the neutralization or paralysis of parallax we use normally to focus on objects; we are required to “relax” our eyes and look into the distance. Neutralizing one parallax allows another parallax to take over; this in

itself proves the existence of *two* parallaxes, one for perspectival space, another for the illusions that will arise by means of projective space's *subtraction* of dimensionality in the *katabasis*.

The emblem of this subtraction is world-famous. It is the single meander of the Thesean Labyrinth, which perplexes many who had thought that all labyrinths involve, by definition, a path where there are choices. This has tempted some commentators to rashly conclude that there are no choices in the Thesean Labyrinth, but nothing could be further from the case. As a one-dimensional projective subspace, the meander of the labyrinth condenses its puzzle to a single issue: "am I going in or out?" This question arises in two circumstances, one created by motion, the other by pausing. In moving through a path that, now, expands then, now, contracts, the sense of inside and outside is eroded. The expectation is to travel through successively smaller circles until the center is reached. But the Thesean Labyrinth plants internal expansion inside its design.



The other side of the question, "am I going in or out," comes if, at any occasion of stopping, the traveler forgets which way he/she was moving. An error is compounded by any attempt made without full confidence. This transfers the question of the labyrinth's external objectivity to the level of the traveler's internal subjectivity. The transfer, leaving the traveler "with no one to blame but him or herself," is like the transfer in *They Live*, from seeing a monstrous world to the realization that one is, one's self, monstrous.

There is, truly, a monster in the labyrinth. The question remaining is: Is the monster the Other, myself, or both?

Escherness of the Binary

It is not entirely stupid to say that the human world involves doubles because we have two hands and two eyes. Immanuel Kant began to worry about this in his inaugural address of 1770, a dry spell in the history of mathematics, just before the Renaissance of projective geometry that revived the work of Girard Desargues and Blaise Pascal.¹⁵ The third dimension which our flat sensing surfaces are able to deduce out of parallax, motion, muscular contractions, and synesthesia is not a single but a double, a *stereognosis*.¹⁶ The concept is about grasp (*propriocept*¹⁷), implicit in the German word for concept as a *grasping*, *Begriff*. We know something when we hold it with two hands; our mind is less certain when we see it with two eyes, but at least parallax can make up for a part of this loss by calibrating distance with a crisscross of the lines of sight that create a stereo sense of depth.

¹⁵ Immanuel Kant, "On the First Ground of the Distinction of Regions in Space," in John Handyside, trans., *Kant's Inaugural Dissertation and Early Writings on Space* (Chicago: Open Court Publishing Company, 1929). See also Section 13 of the *Prolegomena to Any Future Metaphysics*.

¹⁶ I am adopting this term from its limited medical application (*Wikipedia*: "the ability to perceive and recognize the form of an object in the absence of visual and auditory information, by using tactile information to provide cues from texture, size, spatial properties, and temperature, etc.") to include any and all mental appreciation of depth, which always must involve taking a step *past* 2-d pattern perception. Steps involve two feet not just one, so the question is the same as for hands — which will be anointed as the dominant first step?

¹⁷ Ditto the above for "propriocept" (*Wikipedia*: "the *sense* of self-movement, force, and body position"), the subject's awareness of its own body both in relation to itself and to the spaces and objects around it, in relation to touch as a primary sense and tangency as a metaphor for vision and hearing.

There is no object in space that does not imply and require motion, so parallax constitutes a promissory I.O.U. — an immediate short-term loan to be “cashed in” if the viewer has the time to move around in space to verify the three-dimensionality of what seems to be a roundish object but which might be a clever flat stage-prop. With one eye closed it would be almost impossible to tell the difference. With both eyes open, confidence improves thanks to the simulation of movement from side to side that parallax affords. Motion is why we insist that space must be Euclidian, i. e. an X, Y, and Z of width, height, and depth; but motion takes time and energy, which means that we must again insist on using the principle of the Hamiltonian measure, of including all of the sources, and expenditures involved with any idea of a circuit. A circuit is not a circuit without a full inventory, and our inventory of the motion required to confirm Euclid’s XYZ must not ignore the facts of handedness, stereognosis, and propriocept. Central to this collection of data on space’s double nature is the function of the *cut*, the metaphor of splitting space down its middle to create two symmetrical (almost) faces. We qualify the word symmetrical by the idea of chirality, which is both a mirroring and the addition of a thin spatial layer required by the “between” status of the two faces resulting from a cut. This is not a convenient abstraction, but a real thing. Without the “loft” of thin space between molecules of wine and olive oil, there would not be the characteristic tastiness that is the essence of Mediterranean cuisine. Given the attractiveness of foods improved by chirality, we can without exaggeration say that handedness is responsible for billions of Euros, liras, shekels, etc. flowing into the coffers of the tourist trade in this region. No layer between mirror-halves, no tastiness; no tastiness, no hotel, restaurant, or tour bookings. So, this is not an abstraction.

What makes space so “tasty” as a result of the cut between left and right? Notice, please, that a cut *into* something implies that the substances is three-dimensional, but that the symmetrical (chiral) faces that result are exclusively and radically two-dimensional. Also — and this is the critical point — the thin space between the two faces cannot be said to be three-dimensional. On one hand, it is thin but still three-dimensional, but on the other hand its *properties* are exclusively two-dimensional. If it walks 2-d and quacks 2-d, then it’s 2-d. This forces the issue of how 2-d topology interacts with, and sometimes *determines*, what goes on in perspectival space. Generally, this seems to be the case with any and every instance of the uncanny. The appearance is Euclidean but the effectiveness is projective. There is one parallax for presentation purposes but an entirely different parallax protocol for the effectiveness of what is presented and how it is received and understood.

Just as the role of effectiveness cannot be eliminated from any account of “how things actually work in reality,” 2-d topology cannot be eliminated simply because it cannot be pictured. The point is that if topology could be pictured it wouldn’t be topology. This is like Heraclitus’s paradox of the river, whether it can be the “same river” since it’s always in motion. As one wag put it, “a river you could step into twice wouldn’t be much of a river.” The same can be said about the spatiality of Möbius bands, cross-caps, toruses, and Klein bottles. Even though we can picture them, we destroy their 2-d-ness as we do it. We see things — barriers, cuts, twists — that *do not exist in the 2-d space of topology*. Our dimension of observation has converted the non-orientation and self-intersection of the topological form into what seems to us to be a *trap*.

We say that we are trapped when we are forced to do or think or say something. In the famous conundrum of the Cretan Liar, the man from Crete says “All Cretans are liars,” and we are *forced* to go between the two options set in motion by this claim. If he’s telling the truth, then the content of his

statement is false; but, if it's false, then he's a liar. If he's lying, then the content of his statement is true, and then all Cretans *are* in fact liars, but he has just told us something that is true. We are *forced* to circle between these two non-oriented positions. WE are trapped!

In 2-space, the Cretan is not a problem. In fact, the Cretan is telling us that there is a cut between language's status as (1) a speech *action*, a dynamic behavior; and (2) a content, a claim, a statement about something. In French, the difference which is difficult to describe in English is summed up by two related words, *énoncé* (content) and *énonciation* (the speech act). We can't open our mouths without contradicting ourselves by doing two things at once, two things that cannot account for each other. We might even put this mutual self-contradiction into the same form as the *matheme* for the uncanny: A_C/C_A , Action with a kernel of content, in apposition to Content with a kernel of action. Whenever we speak we contradict ourselves, so the Cretan is no stranger. He simply intensifies and localizes what is generally true of all speech, everywhere, anytime.

Is this a kind of linguistic left and right hand, a kind of spoken parallax? By extension, is parallax itself a version of the A_C/C_A by which we speak of the world? Can we say of Euclid and Projectivity that there is a formula of E_P/P_E , where 2-d projective forms are the kernel of full Euclidean three-dimensionality and, for Projective space there is a Euclidean kernel? Certainly Pappus of Alexandria would have argued for this. There is the paradox that, although he discovered and articulated the principles of projective space in 300 a.d., long after Euclid's works had been accepted as the master plan for all spatiality, he also discovered that projective geometry was logically foundational for Euclid. This was evident particularly in Euclid's controversial Fourth Postulate about parallel lines. The infinity at which two lines at a right angle to a common line will converge only in a "fictional" way at infinity was problematic. It was not clear that it was a real postulate with the same logical force as the other postulates.



Pietro Perugino, *Christ Giving the Keys to St. Peter* (1481-82)

In projective geometry, perception is taken to be real, or as Lacan would write it, Real. It was Real and also "impossible" in the sense that the Real cannot be symbolized. Attempts to symbolize it meet with resistance, and the resistance is evident in things like paradoxes, puzzles, and traps. In other words, the things that interest us and engage our imaginations are none other than attempts to domesticate the Real, which by definition resists domestication. Trying and failing to domesticate is the territory of the uncanny.

In perception, we actually see lines converge on the vanishing point at infinity. Where else can they go, but to the point on the horizon that we see clearly as their linear destiny? In

Renaissance painting, this point is embellished as a temple, or specifically a *tempietto*, the building that celebrates the apostolic virtues of the point. The point is not simply a dimensionless feature of geometry but an element specifically able to *convey a message* from a source designated as supernatural. This geometrical *conatus* has traditionally accompanied a legal idea of authentication.¹⁸ This is complex to explain philosophically but easy to portray in architecture and painting, where the vanishing point's role in

¹⁸ Tommaso Parducci, "Sul concetto di conatus nella filosofia di Vico," *Laboratorio dell'ISPF*. 2018, vol. XV (9). DOI: 10.12862/Lab18PRT.

projective geometry is assumed as a given and portrayed directly, without any Euclidean fussiness about the impossibility of infinity. There, in front of us, is the vanishing point, with its family of parallel lines meeting at a *templum*, whose etymology tells us is primarily a *distinction*, a *cut* (Gr. $\sqrt{\text{TEM}}$, τέμ-ν-ειν, fut. τεμῶ).¹⁹ The cut is what, Perugino's painting tells us, makes the vanishing point's *conatus*, its temple aspect, into a center, a nexus through which all lines must pass.

The mathematics of the *vesica pisces* principle that the periphery's extremity is simultaneously a central pivot is complicated and beyond my abilities to understand, let alone explain. However, it is easy to explain through a film example. In Alfred Hitchcock's 1946 spy thriller *Notorious*, there is a scene involving the theft of a key. The way the key functions in the plot, and the way Hitchcock films and edits the scenes to depict the key's (literally) *key* role, can help us understand the Escher-ness of the uncanny, language, and geometry, all at the same time! Beginning with A_D/D_A of the uncanny, the A_C/C_A of language, and the E_P/P_E of geometry, the complex mirroring function is foundational to the "temple function" of the vanishing point itself, its function as center and periphery, C_P/P_C . This fourth Escher sentence takes us into a mathematics that is "actionable" in art, literature, architecture, and film. It is the means by which "psyche is extended," and its mind-bending Escherness guarantees that we can never "know anything of it."

In *Notorious*, an American spy, T. R. Devlin, has recruited the daughter of a convicted Nazi spy, Alicia Huberman, to marry a German industrialist living in Rio de Janeiro, Alex Sebastian. She has discovered that something important has been hidden in Sebastian's wine cellar, and she and Devlin must take advantage of a party at Sebastian's mansion to investigate. Alicia manages to steal the wine-cellar key from Sebastian's key-chain, but the couple must get to the cellar before Sebastian discovers that it's missing. They realize that their window of opportunity is limited by the champagne supply. If guests drink up the supply

at the bar table, the butler will have to ask Sebastian for the key to get more bottles. Sebastian will then notice that the key is missing and realize his wife's complicity.



Hitchcock, in his standard cameo appearance, guzzles down a glass of champagne as if to underscore his technique of turning wine into blood, potential blood at least, if Sebastian discovers the theft of his key to the wine cellar. This is a true *pharmakon*, both elixir and poison, showing the affinity to the Escher-formula embodied by the uncanny's polarity expression, A_D/D_A , expanded to describe language, geometry, and architecture's identification of the center and periphery in the *templum*.

This complicated situation is made readily understandable by Hitchcock's story-in-a-story, the "tale of the key." He uses a series of close-ups to show how Alicia stealthily removes the key from the key-chain while Sebastian is not looking; then how she holds it in her hand to pass it to Devlin as she greets guests in the foyer of the mansion — the famous boom shoot where the camera physically zooms in on a detail of her clinched hand. At the pass-off of the key to Devlin, he remarks that everything will be fine unless they run out of champagne, and Alicia gasps to see the guests enjoying their drinking in a new, perilous way. Hitchcock follows her new re-adjusted point of view by showing guests as greedy guzzlers. Each glass they consume limits the time Alicia and Devlin have to get to the wine-cellar.

The audience readily understands this logic of film story-telling. Champagne has flipped, from being an elixir to a poison, adding a fifth dyad to our list of Escher expressions: E_P/P_E . The film version,

¹⁹ Walter W. Skeat, "temple," *An Etymological Dictionary of the English Language* (Oxford: The Clarendon Press), 634.

however, helps us understand the role of the cut, the *templum*, in relation to the need to convert audience's normal sympathetic enjoyment of the party scene to an anxiety-mode running parallel to Alicia and Devlin's. This is absolutely necessary to follow the story, but the quick turn-about of an audience from one extreme to another is not an easy matter. Such a turn is about effectiveness, and the trick of the turn depends on a Hamiltonian understanding of the visual energies governing framing and latency.

The question of insulation is critical. If you have all the space in the world, or all the time, you don't need it. It's only when the wolf is at the door, or death, that you need to hold evil at bay. Space and time have let you down; their extension has accorded; the markers have moved forward and are unexpectedly close. This is the source and cause of anxiety. What is felt is the result of the moving of spatial and temporal *markers*: "over-proximity." Anxiety leads Romulus to build a wall around a precinct, but it has a double function. People must be able to come and go but danger must be filtered out. This is the double function of the portal: to allow passage of the body while restricting that of the spiritual threat.

Enfiladed hill forts of the Iron Age were designed around the same principle. Enfiladed entry required any who would be admitted to the interior to be subjected to interrogation (also threats and taunts) from defenders standing on hills above the entry trench. The elongated entry-way was a means of increasing spatial extent within a compact radius. The "truth function" of the defenders' interrogation is evident. The extension of the path created a trap at the same time it stretched out the time of entry. Entrants could not easily reverse course to escape an unsuccessful test. A new force arose from this latent energy source: an "eigenvector" that would, from the literal cut into the earth that was wound around a central protected space, be a spiritual filter, a distinction between authentic and deceptive visits.

Alicia and Devlin's energy of anxiety comes into focus and becomes sharable by the audience because they have entered the enfiladed path. Their descent into the wine-cellar becomes, like every *katabasis*, a quest for truth. They need to find what is hidden "among" the wine bottles but they are uncertain about the preposition. Is it something "about" the bottles, "within" or "among" them, or actually "inside"? Devlin accidentally knocks a bottle over the shelf, it smashes on the floor. Hastily trying to clean up the mess, he discovers that a grainy substance has been concealed within the liquid — uranium! He hastily replaces the bottle but gets the position wrong — another involvement of ambiguous prepositions, and this misplacement will provide the coming scenes with their efficient cause. For this moment, however, the accidental discovery will pull the time marker forward. The couple will have to find a way to justify their presence in the basement, exempting the wine-cellar from suspicion. They embrace near the external doorway just as Sebastian comes down the stairs. Are Alicia and Devlin in love? Originally Devlin had cynically recruited Alicia as a drunk who was nonetheless disgusted by her father's treason. They fell in love once they worked together in Brazil to infiltrate Sebastian's network, but Devlin regarded Alicia's marriage to Sebastian as being over-effective, even though it was the only way to gain full access to the Nazis plans. His cynicism returned but hung in the balance as they found the secret they had been looking for. So, was this kiss a "kiss" or a *kiss*? As a cover for their presence in the basement, it was meant to appear

authentic to Sebastian, but as a ruse it was inauthentic. However, the backstory makes it doubly ambiguous. It was a “kiss” made to look like a kiss that was really a *kiss*.²⁰

Is the mystery of “psyche extended” solved by this example of Escherness, whose logic can be reverse-engineered to unravel the complexity of language’s *énoncé/énonciation* components, geometry’s projective ambiguation of center and periphery, and architecture’s temple paradox, the conflation of the temple and the labyrinth? Is the Escherness of Escher sufficient to say that psyche’s extension is contronymic both in terms of time and space *and* knowing/unknowing (*kenosis*)? If psyche is indeed extended, then it must be allowed that we think forward to the next and most telling conjecture, that psyche is *extension itself*. This must be considered chirally, as a chiasmus. If psyche is extension, extension must be psyche.

If this mirror conjecture is true, then we must construct new critical expressions to come to terms with this internal symmetry that is simultaneously an exteriorization. From the power of the point, *conatus*, we must include the function of the mirror to “ambiguate” the viewer and the viewpoint. Fortunately, this word already exists — “cathetus” (from geometry and Brunelleschi) — but to mark its extended meaning in relation to *conatus* it should transmute into something more general: “cathesis.” For the effect on parallax, where this internal symmetry is analogous to the stereogram, the perspective that emerges from within the normal parallax-induced perspective, the edge of the figure becomes critical and we should borrow a term from pottery decoration — isomeric — that describes how a boundary can indicate the separation of something from nothing. The effect of this boundary echoes the ancient practice of friends breaking a ceramic token into two parts, hopefully to be reunited along the authentic fracture line, at their reunion: *tesseræ* (a term also used by the literary critic Harold Bloom).

The issue of circuits, cycles, and completion is evident in all of the X_Y/Y_X -style expressions, so the paradox summarized by the French saying *plus ça change, plus c’est la même chose* (the more things change the more they are the same), should be used to finish off the idea of *conatus* as both change and permanence. This can be taken from Freud’s essay on the subject, *Transience*. Although experience encounters only “section views” of the cycles of growth and/or decay, they are both discredited by the reality of the full cycle (the Hamiltonian) but resist and remain as the only possible evidence. This section slice of contingency and momentary immediacy should also have a special name: the *katagraphic cut*. This is also the “/” of the X_Y/Y_X but it is not abstract. The katagraphic mark is something that has a history, and this history is consistent with its logical and *psycho*-logical function. The katagraphic mark is documented by Jesus’s famous use of it in the story of the woman accused of adultery (*John* 8, 1–11; Causse 2013: 113–4) and by Livy’s story (*Ab Urbe Condita* 45, 12; Livy 1951: 43–5) of Gaius Popilius Laenas, the Envoy of Rome (168 BCE) who persuaded Antiochus IV Epiphanes, the King of Syria, to withdraw from Egypt by drawing a circle in the sand around the would-be invader (Lacan 2011a: 65). Vico underscored the importance of the superficial mark by reminding us that, for the ancients, Hades was as close as the bottom of the furrow (Vico 2016, §546).

²⁰ In an earlier Marx Brothers film, *Animal Crackers* (1930), Groucho questions a guest at Mrs. Rittenhouse’s party: “Say, you look like someone I know — Emanuel Ravelli.” “But,” the guest replies, “I *am* Emanuel Ravelli!” Groucho, nonplussed, responds, “Well, then, it’s no wonder that you look so much like him — you are both alike!” The doubling and internal misrecognition of the two Ravelli’s, the one who thinks, along the lines of $A=A$, that he is identical to himself, cut from the Ravelli who is a deception to begin with, someone who can “look like himself.” This goes to the heart of the Russell Paradox, the \notin of the set that is and isn’t a member of itself.

In architecture, there is an even more important role played by the katagraphic mark: the foundation ritual of cities. In the justifiably famous story of Romulus and Remus (whose twinship justifies an initial comparison to the “twins” of A_D/D_A), Romulus first plows the ground to make the initial distinction between inside and outside. But, this turns out not to be the “inside” of the ring from the “outside” of the space outside the ring, but from the inside of the outside and the outside of the inside. The space of the plow line was preserved and forbidden to be covered by walls or buildings. It was the *pomærium*, the *pro muram*, a line that divided the city “from itself.” Ancient Roman law specified that the *pomærium* in fact was the *only* space regarded as the true Rome. The spaces on either side, the interior inside the city walls and the space outside, were both relegated as “territories.” It is a historical fact that the only true Rome was a torus — not the “immersed” versions of the torus we know well through bicycle tires, bagels, and doughnuts, but the 2-d projective surface, the surface that provoked Remus to mock the furrow by jumping over it and the surface that provoked Romulus to kill him for this act of impiety.

What is lost to history is that this murder of twin by twin was a positive act, akin to the rotating relation of the Dioscuri, Castor and Pollux. Rotating kingship served European culture from the late Ice Age on to the emergence of city-states. Kings were fictionally or actually sacrificed on a fixed schedule to insure the integrity of the city and justify the consolidation of family hearth-based religions into a single civic flame attended by virginal guardians. The collective *manes* did for the city what the household *manes*, the penates and lares, did for the family’s space: the insulated it. They established the katagraphic cut as the principle of allowing secular access while preserving religious isolation. The separate customs needed to structure the political, religious, and social foundation for this contronymic urban principle were indeed uncanny in modern eyes, where full secularization obscured the metaphoric logic of the cut, the furrow that was so close to the principle of burial.

If the katagraphic mark helps us understand the dynamics of transience, the new critical terms will establish a means for scholarship to move “psyche is extended” to “psyche *is* extension.” The move will be necessarily grounded on ethnological evidence — how cultures establish a collective unconscious out of customs, rituals, protocols, and the built evidence that they know exactly what a trap is and how it works, by combining conductivity and insulation.²¹



Anastazi trickster

²¹ The technical name for this is “a non-affine transformation,” or, “non-affinity facing.” Alireza Moharrer, personal conversation, December 21, 2002. Affine geometry is what remains of Euclidean geometry if measures of distance and angles are subtracted. Adding a point at infinity and line at infinity to the affine plane allows a projective plane to be constructed. A projective plane is the Hamiltonian of Euclidean space.