# Parallax and the Ames Window One, Two, or One of Two Minds? 

Don Kunze



Figure 1. Master of the Apollo and Daphne Legend, active 1480-1510, "Daphne Fleeing from Apollo." David and Alfred Smart Museum of Art, University of Chicago.

We look at the world, the world looks at us. We have a good, geometric way of defining the way we look at the world, it's a combination of optics, contractions of muscles of the eyes and lens, the somewhat mystifying overlap between two views of those with binocular vision, and other cues that allow us to flesh out what is not given directly: the depth dimension. We corroborate visual and haptic data with other cues to judge distances, for the very good reason that threats and attractions depend equally on our ability to gauge how far away they are, in case we need to run away, fight, or chase. Before there was agriculture, there were thousands and thousands of years spent hunting and gathering, and the protocols associated with these activities became etched into the human psyche, even when it turned to plowing fields and sitting at desks.

Parallax could be stripped down to a diagram showing how two eyes use geometry to gauge distance, but this is a greatly simplified model. Nonetheless, it accords with other depth-devising constructions, such as perspective drawing, photography, and surveying. Without some means of measuring distance, civilization would quickly grind to a halt. The upside, that no canons or missiles could be fired, would be overwhelmed with downsides. Parallax is an essential way of being in the world.


Figure 2. Schematic statement of the two versions of straightness (orthos). The circle is the concentric or gravitational model, the tangent line is the "beam" theory. The concentric schema wins in tests involving falling or flying, the beam theory wins in flight contests. Orthos was the name of the appropriately two-headed dog protecting Hades. The two themes conflict in the story of Daphne and Apollo, initiated by Eros's double arrow of Love and Hate, leading to Apollo's chase and Daphne's flight, then Daphne's paralysis at the point of her conversion into a laurel tree.

Like the parallax optical diagram, however, it's quite easy to settle for the apparent clarity offered by the Euclidean model of space as an XYZ affair. How could that not be the case? There seems to be no disagreement that our sense membranes and tissues are basically two-dimensional. How else might they mediate the world "out there" with the nervous system that bundles and channels sensations to the brain. The complication is that the body's chief sensor of touch, heat and cold, and radiant warmth is a continuous surface that forms a torus, the means of ingestion that brings the outside inside and then, after digestion has had its fill, leaves it for waste management systems to deal with. This folding of a sense organ complicates our idea of inside and outside, since what is swallowed simply goes to another kind of outside.

Another complication is about what is meant by twodimensional. There is clearly some penetration of sensation into cells, some intermingling of energy received and energy relayed. Also, while the curvature of the retinas means that no "straight line" in nature is perceived materially as such. The spherical relation of the viewer to the straight line must be replaced with a concept of a straight line. What happens on the retinas must be "thought-into" being what it will later prove to be, if we test it.

The tests for straightness varies, and proves this contradictory fact. There are two versions. There is the sagittal straightness of the taught wire, the eye-beam, the look. Then, there is the spherical straightness of sea-level, water in a lake, the horizon as abstract. The two types interact without much commentary. Imagine a large basketball court made flat by the sagittal method. A marble on its surface would roll to the center. The model for this would be a line tangent to a circle, its ends raised above and center depressed in relation to the system of gravitational field, which is concentric, because the horizon system is based on distance (Fig. 2). At this exaggerated scale, it is clear why the marble would roll to the center of the court, or the arrow would fly in a curve.

The two models of straightness provoked interest and produced poetic puzzles. In the story of Daphne and Apollo, Eros, insulted by Apollo's mockery of his archery skills, crafted a bidirectional arrow able to strike the god and nymph in a totalizing way, which is to say that one vector charged the space in which Apollo would chase, following sagittal logic, and Daphne would flee but be trapped by the rules of concentric gravity (Fig. 1). The story could be told in five
words: one flew, the other fell. Apollo chase cannot defeat Daphne's chastity. ${ }^{1}$ The story is about geometry as much as it is about mockery, and it should suggest that the equation is reversible, that the conflict between the two types of straightness can always revert to the interpersonal conflict provoked by insult: the intentional humiliation of one subject by a presumed equal, and the reciprocal submission of the other to the humiliation of the one. The former is the global condition, the latter the local application.

If humiliation is the local experience of the global condition, our "equation" linking the two models is the unavoidable spatial-temporal contronym that is the conclusion of the ApolloDaphne fable. The otherwise inexplicable paralysis of Daphne, which would trap her as a human form but give her honor as long as she metamorphs into not just any tree but a laurel, the tree whose branches are used to decorate the brows of heroes, converts the physics of the equation into the language of humiliation and respect, the "human side of the story." In other words, characters such as Daphne and Apollo chase in one system and fall in another, because the two systems have been running in parallel all the time. It is not for nothing that the dog guarding the gates of Hades had two heads and was named Orthos.

The logic is not hard to grasp, and not limited to the poetic consciousness of antiquity. Ted Kooser writes, in his poem " 5 p.m.". The pigeon flies to her resting place / on a window ledge above the traffic, / and her shadow, which cannot fly, climbs / swiftly over the bricks to meet her there. ${ }^{2}$ Marcel Proust used the shadows of birds across the cathedral wall as an epiphany in $\grave{A}$ la reserche $d u$ Temps Perdu. In other words, this idea is not isolated by history, culture, or narrative circumstances. It is a universal, a logical construct, a component of collective consciousness and, therefore, worth our attention.

Thus, it is not for nothing that in Seminar VII, The Ethics of Psychoanalysis, when Lacan mentions architecture specifically for the first time, that he cites the Apollo-Daphne story and identifies the nymph's paralysis with the "surface of pain" that is explicitly architecture. Not architectur-al but architecture. What poets have known about since antiquity and what modern poets such as Kooser or Proust have readily employed for their own constructs of epiphany is also what Lacan knows in 1959 but acknowledges officially only in 1961, thanks to a footnote added to a text in 1965 but possibly has forgotten that he we first bitten by Orthos in 1935.

Lacan, Ovid, Proust, and Kooser walk into a bar ... what's the rest of the joke? The joke is that "they cross it." The bar separates the signifier from the signified and is the basis of the Symbolic in Lacan's RSI system, graphically rendered as a Borromeo knot, where any two rings are held together by a third, which can be said to be either present (applying a force) or absent (not physically connected). The thesis is related to the riddle of parallax. Is it the straight line of flight,

[^0]${ }^{2}$ Ted Kooser, "5 p.m." Flying at Night: Poems 1965-1985 (Pittsburgh: University of Pittsburgh, 2008).
where a diagram can be drawn in a Euclidean way to show the binocular overlap, or is it a gravityinduced fall that holds in place "she who would remain chaste not chased." The two versions conflict, yet it is hard to find one without the other, as the Apollo-Daphne story suggests. They are a spatio-temporal contronym. ${ }^{3}$ Can we say that the Euclidean model works for us looking at the world, while the concentric gravity model works for the world looking at us?

Lacan's assertion that the topology of the pure projective plane is nothing less than the Real not a picture of the Real (pictures are impossible in projective geometry), not a model of the Real (i. e. not a template), and not an analogy (another kind of model), but T-H-E R-E-A-L itself, ipsum. He means this the way Vico asserted that the made was the true, and vice versa. Verum ipsum factum. The true and the real are "convertible," meaning that, like the language of flying and falling, though seeming opposites, are two (antipodal) parts of the same machine of straightness. The parallax of looking-at requires there to be another parallax, one that situates Daphne's paralysis in its own physics of honor. If Lacan says this is the Real, it's the Real of projectivity that allows both for the closed curved surface that was Daphne's trap, assembled in the simultaneity of the moment she thought to flee, and for the love of Apollo, inflamed by Eros's double-projectile of lovehate.

We had better use the proper name for this messenger of antinomous passion: a onedimensional sub-space, which in projective geometry is a vector that passes through a single point, with all other vectors in space. It had originally been one of a family of lines meeting at a vanishing point; now it passes through that vanishing point to create a system of concentricities measure by the real projective plane (Fig. 3). As perplexing as this illustration may appear at this point, it sums up the mutual antagonism of the two systems of parallax (distance) in terms of the


Figure 3. Projective geometry converts the horizon as a limit of the visible to a point through which vectors ("twodimensional sub-spaces") pass through. The point then becomes the origin of a Cartesian XYZ space, where a plane positioned at a unit distance above the X axis will simplify the description of the vectors by describing them as two coordinates rather than three. The role of the 1 is the same as that employed by Lacan to describe the Golden Ratio as the infinite depth created by the powers of $a$ in relation to the A, Autre, in his slide-rule analogy in Seminar XIV, The Logic of Phantasy. It is the 1 that assures that the sum of $a$ and $a^{n}$ will "oscillate" left to right within its unary space, approaching the value of $\emptyset$, which on the slide-rule was represented by $1 / \mathrm{a}$.

[^1]stasis of projectivity, which none can see, and the dynamics of the chase, to which anyone can assign motives, calculate who will win and lose the race, or be a lover or hating beloved.

## Projective Geometry

Jacques Lacan was specific. Projective geometry was nothing less than the Real, the Real itself, the Real as that which resists the Symbolic as steadfastly as Daphne resisted Apollo's advances. In fact, this is why Lacan found, in Ovid's story, a way to describe the contronymic composite of projectivity, its "global" principle of gravity versus its "local" principle of flight, each corresponding to its own system of parallax. The Real was neither the global nor the local, but the bar separating the two, the bar of the horizon that converted to the hollow point that was also the vectors passing through that bar on their way from visibility to invisibility. ${ }^{4}$

The problem for projective geometry in general has always been its relation to Euclidean geometry. It was discovered after Euclid had written his six books, by, coincidentally, a mathematician who had written a commentary on those same six books, Pappus of Alexandria. However, when Pappus devised his theorem of projective geometry, he realized that this theorem described a geometry that was not only wholly different from Euclid, but logically prior. Priority could mean only one thing: that projective geometry was embedded within the "everyday dimensions," the XYZ of Euclid, but as a principle of effectiveness, a principle that allowed Euclid's vectors, curves, distances, and angles to "happen," but which was also the eventual limit where these measures could go no further: the Fifth Postulate. Euclid would be forced to invent a fiction about infinity in order to explain why parallel lines did not meet at the vanishing point. This infinity would be based on a proof about equal angles of parallel lines intersected by a third line cutting through both.

Of course, Euclid did not live in the real world. He lived in a world of demonstration, where there was only one version of straightness, where the horizon was not circular, where gravity would not pull a marble to the center of a flat plane, nor where the brain would not be required to "think of straight" where it saw, plainly before it, a curve. Euclid, it is safe to say, was thoroughly perplexed at why Daphne should become a tree just because Apollo had insulted Eros. There should be no blame cast upon Euclid, however. We do not live in his space, but we enjoy it as a "space of demonstration," where from one false principle (straightness as axiomatic), an entire

[^2]\[

$$
\begin{aligned}
& F_{\text {gravity }}=m g \\
& F=G \frac{m_{1} m_{2}}{r^{2}}
\end{aligned}
$$
\]

Figure 2. The force of gravitational attraction (above), generalized (below) is the structure of two- and threedimensional concentricity, by which objects fall to the center when they fail to resist and flee from the center when fright compels them to run. The decision between fight or flight lies with the parasympathetic nerve system, which at the level of physics is set at a hypothetical trigger point that "floats" or is held in orbit. The moon in its orbit of earth could be called passive paralysis, since the moon neither flees nor falls but is held in position, defined by the mensual cycle, with universal implications variously interpreted by each culture.
system can develop: ex falso quodlibet sequitur ("from the acceptance of a false principle, anything is possible"). Lacan, realizing that Euclid's mistake was on the level of Russell's paradox about a set being able or not able to join itself (the same principle as Lacan cited for all men, $\forall \mathrm{x} \varnothing \mathrm{x}, \exists \mathrm{x} \sim \varnothing \mathrm{x}$, "all those who would call themselves men - "who would claim the honor of being called men" - do so at the risk of being humiliated by "at least one" who does not obey the phallic rule). If anything confirms Russell's Paradox, it is this matheme. Insult impacts directly on the rise or fall of sexuation, and without this Daphne would not be compelled to flee and then face the logical trap of paralysis within the 2 -space of her own specially devised parallax.

To demonstrate as Euclid did, inventing his own operational space at the same time, was a secularization of de-monstrare, not a removal of paradox as the "monster" is disassembled, but an action de monstrare, about the monstrous, about the metonymy of temporality projected as a composite but problematic figure with a head of a goat representing spring, a body of a lion representing summer, and the tail of the serpent to entail winter. The Sphinx herself presented this same meroic model in her riddle presented to CEdipus, not guessing that he would know the answer which would be equivalent to the password gaining him entrance into Thebes, but that this very password would further extend his blindness to the prophetic logic connecting his birth, abandonment, survival, patricide, and future incest into a single paradigm.

This is monstrosity of blindness to one thing and the totalizing perspective of Tiresias, who after seeing snakes copulate was, like others of his "clan," ${ }^{5}$ the Semitic-speaking peoples who moved from Syria into Cadmeia towards the end of the second millennium, b.c.e. Tiresias's principle is entirely parallaxian. The sight of the copulating serpents was an identity, a case of the view looking back at the viewer with an extromissive power-ray that converted the viewer into

[^3]

Figure 4. Ouranos. Central part of a large floor mosaic, from a Roman villa in Sentinum (Marche, Italy), ca. 200-250 C.E. Uranos, in the form ofAion, the god of eternity, is standing inside a celestial sphere decorated with zodiac signs, in between a green tree and a bare tree (summer and winter, respectively). Sitting in front of him is the mother-earth goddess, Tellus (the Roman counterpart of Gaia) with her four children, who possibly represent the four seasons. This is a projective space, in that Uranos, who is outside the visible world (Gaia, earth, is shown inside the zodiac belt.
"what he was, 'for Real", which in Tiresias's case was a man-woman, the single body in-corp-orating the exclusionary principle of male identity and the not-all principle of the feminine. In parallax terms this was the "one-dimensional subspace," the projective (extromissive) vector becoming "what it always was," a panoptical (horizonal) visuality, capable of prophecy.

Tiresias points us to two main concerns in the connection of ordinary parallax with what we come to identify as a second parallax lying inside this first parallax. The theoretical issue has to do with the fact that the second parallax is actually logically primary, and even foundational, for what we take to be the parallax of everyday reality. This is the way that Tiresias, in seeing the copulating snakes (historically second to his life as a man), was then extromissively seen as what he was primarily, as a race that sprang from the earth, which the serpents presented as a composite of sexual coupling. Tiresias became a man-woman "secondarily" because he was a man-woman "primarily," thanks to his being a member of the Cadmus family. The "dragon's seeds" were sown by the penetration of the earth (Gaia) by the sky (Uranos). The same fate applied to projective geometry, which although it was discovered "secondarily" to the elaboration of Euclid, it was discovered to be prior to Euclid, a logical foundation. This retroaction is the means by which a basis, to be a basis, is concealed by its progeny, who are "destroyed" upon the discovery of their true father (Saturn devouring his children). ${ }^{6}$ In Vico's account of the origins of humanity itself, this sowing of the earth by the heavens was the lightning that ignited fires, accompanied by thunder that was regarded as the Word of Jove.

This cosmic act of sewing could mean only one thing to the first humans: the fire was the point phenomenon of prophecy (il vero of Vico's explanation of myth as vera narratio), and the hearth that concentrically surrounded it was the site of divination, supervised by Hestia and her daughters, whose allegiance was assured by the "fact" of this localization, that the fire was an

[^4]extromissive power that compressed the "horizonal" and panoptical truth into a single vector that was orthographic and orthological: corrective and universally surveillant. ${ }^{7}$

There are two or three things one can know and then know a lot of other things. The story of Tiresias is one of those two or three things. It explains for example why the science of divination developed by the first human societies in awe of the thunder's word ${ }^{8}$ required (1) a principle of paralysis (Prometheus, chained to the rock while his liver, the preferred organ of divination, was repeatedly devoured by the Jovian eagle then regenerated - truly a case of ha fegato! as an expression meaning "he has guts!"); (2) a protocol by means of which the "priestesses of Hestia" had to break their vows to marry exogamously; and (3) how and why the optical principle of the "one eye" - meaning the eye of the forest opened up to a view of the sidereal sky - was correlated to extromission in relation to this necessity to break with "promethean" rule of paralysis. ${ }^{9}$ The first humans were aware of metonymy but not metaphor. Metaphor was the principle of suppression, by which the first humans viewed the world as a self-image without realizing this transposition. ${ }^{10}$ Metonymy involves the distinction of a "figure," a meaning or physical shape that "stands out," from a "ground," details that have been "suppressed" to a level of less significance, physically modeled by the visual figure-ground's imagined plane of the background. The key idea here is that the background is always the background. Like the visible world's horizon, it recedes to be the permanent limit of the observer in any position. The
${ }^{7}$ Culture begins with the clearing of the forest that was later called the Nemean Lion, a monstrous tri-partite form that qualified it, also, as a dragon. The dragon's "teeth" were the plow-shears of the first agriculturalists, personified by Cadmus, who "sewed the dragon's teeth," meaning that he cultivated the land with ploughs, whose triangular shape led to this comparison. Giambattista Vico, Universal Right, trans. Giorgio Pinto and Margaret Diehl (Amsterdam and Atlanta, GA: Rodopi, 2000), 478. Note also that the curved part of the plough was the urbs, connecting it to foundation rites where the pomœrium is plowed and subsequent building on the furrow prohibited.
${ }^{8}$ This awe is historically and ethnologically durable. See John Nance, The Gentle Tasaday: A Stone-Age People in the Philippine Rain Forest (New York: Harcourt Brace Jovanovich, 1975). Donald Verene has pointed out the "Vichian" passage in this book, where the Tasaday report that they are terrorized most by the one thing in the forest that they can't see, the thunder. Vico's Science of Imagination (Ithaca: Cornell University, 1981), 89-90.
${ }^{9}$ Extromission is associated with a "line of fire" that is simultaneously truthful, pure, and terrifying. Even in the 1951 film, The Day the Earth Stood Still, this logic is obediently applied. The robotic security guard Gort can melt weapons of the soldiers without hurting the soldiers themselves. His beam is the condensation of the circle of star-power (the space ship represents the confederation of all the galaxies), repeating the template of the circle and the line, a schematic of the Ames Window Illusion. In the Ames Window, the ruler balanced on the window ledge is orthogonal to the window, serving the same function as a "cathetic line" drawn perpendicular to the flat plane tangent to the earth's sphere. The earth, in rotating, imitates the Ames Window, the "spectacular surprise" comes when, along the line of cathesis, we bear witness to the collision of the two logics, chiral oscillation and $360^{\circ}$ rotation. Thus the second parallax emerges from the first.
${ }^{10}$ The first humans were "giants," according to Vico, because the Greek word, $\gamma$ i $\gamma \alpha v \tau \alpha \varsigma$, first meant "children of the earth," i. e. the race of Cadmus, borne of dragon's teeth, the auspices of the hearth. Vico, New Science, $\$ 13$. Ignorant of metaphor, metaphor constituted the logic of the first human conception of reality, metonymy was the structure of the transactions between humans and the messages divined through techniques of auspices. The flame connected the first families, at the locale of the hearth, to the manes, or ancestral spirits. Numa Denis Fustel de Coulanges, The Ancient City: A Study of the Religion, Laws, and Institutions of Greece and Rome, trans. Willard Small (Mineola, NY: Dover Publications, 2006).
background cannot be visited; as each part of it is conscripted to be a part of con-figur-ation, the background is re-supplied on a one-to-one basis as we change our positions as viewers able to move around.

Metonymy's effectiveness is based on the utility of its prominent feature, as in the case where the sails of ships or heads of cattle allow them to be more easily counted. A metonymized world produces various kinds of utility, the most useful of which is parallax measure, where the cry of an animal or crash of a fall allows the source to be located and estimated. A field comprising metonymies can be surveilled; its threats identified and located; its resources located and tracked. It is Apollo's world, the world of the chase, improved by enhancing the visibility of the prey and invisibility of the hunter or, from the hunted's point of view, the reverse: camouflage and early warning systems. ${ }^{11}$

The hunter-gather's world is fundamentally metonymical because of the practical need to divide figure from ground, the utility of parallax in both optical and indirect geometrical terms involving triangulation, short-cuts, camouflage, and tracking maneuvers. The point is not to generalize the particulars of pre-historical perceptual practices but, rather, to show the role of the circuit to the beam, the horizon to the central point, the rotation to the oscillation. This is the geometry that, in projective topology, allows the circle of the visible to invert to a hollow point through which vector that had previously aimed at points on its periphery now pass through it in a common bundle, vectors that can be given Y and Z values as they pass through a plane suspended as a $Y$ unit placed at 1 above the X axis, leaving only X and Z to be accounted for. ${ }^{12}$ From this same plane, spatial forms of the cross-cap, Möbius band, torus, etc. can be described two-dimensionally but seen only if they are "immersed" into a space that allows the twodimensional plane to be viewed at a distance, the Euclidean "space of demonstration." From the circle to the point and the point to an extromissive beam, projective figures appear before out eyes, to dazzle us with their properties of non-orientability and self-intersection. We enjoy these

[^5]abstractly, but we are amazed when we find that this sequence from circle to point to line of sight is also the protocol of stories to be found in ancient myths, where themes of paralysis, monocularity, and vows broken through charade carry forward ancient practices that were the basis of "cyclopean" cultures incapable of knowing that they were thinking and seeing thanks to their denial of the metaphor by which the world had been remade in their own image. The metonymies of the chase, elaborated in the ritualized warfare of "heroic" peoples, were no less obedient to these three principles, principles that could and still can be "told" through simple diagrams showing the relation of circles to points to planes to planes in perspective. From Jove to Tiresias to the modern believer in the literal, the geometry has been the same. It is a geometry to be witnessed more than understood, since its wonder lies at the neural level where two parallaxes overlap, where our perception and neural processes allow us to create perspectives inside perspectives, even at the expense of logical expectation.

## The Ames Window Experiment



Figure 5. Maquette used in the Ames Window demonstration, a trapezoid made to appear, when viewed faceon, as a window in perspective. When the flat image is rotated, the perspective appearance does not chance, and although each edge takes turns being in front of the viewer, it appears that only one edge oscillates from left to right. Thus, an object penetrating a pane of the window, which is perceived to revolve $360^{\circ}$, seems to violate the space of the window at each $180^{\circ}$ position. Video from Dr. Deane Hutton, The Curiosity Show.

The orthographic has two meanings. First, it is the idea of a correct relationship, based on the root term orthos. Second, it is used to designate a right angle, an angle at ninety degrees, a "cathesis line," so it's also about the possibility of a Z dimension in the space created by an X and a Y , a projective vector that connects the viewer with the viewed. ${ }^{13}$ We can hardly imagine a condition where we could see something that we couldn't really see, but the Ames window experiment subtracts from what we think we see. It is about what happens when the Z dimension is not allowed to have its usual freedom of movement, which is the chance to see if a three $D$ thing is really three $D$.

There are two main concerns in the connection of ordinary parallax with what we come to identify as a second parallax lying inside this first parallax. The theoretical issue has to do with the fact that the second parallax is actually logically primary, and even foundational, for what we take to be the parallax of everyday reality. The orthographic has two meanings. First, it is the idea of a correct relationship, based on the root term orthos. Second, it is used to designate a right angle, an angle at ninety degrees, so it's also about the possibility of a Z dimension in the

[^6]space created by an X and a Y , a projective vector that connects the viewer with the viewed. We can hardly imagine a condition where we could see something that we couldn't really see, but the Ames window experiment subtracts from what we think we see. It is about what happens when the Z dimension is not allowed to have its usual freedom of movement, which is the chance to see if a three D thing is really three D .

A trapezoid can be dressed up to look like window in perspective so that, when we see it from either side, either edge looks like the edge that is closest to us. This converts a rotation of the trapezoid appear to be a back and forth motion instead of a rotation, and we can play with this effect by placing an object running through one of the window-panes that will appear to revolve rather than move back and forth. The effect is particularly strong when we see the trapezoid endon, when we think we are seeing the same edge but in fact are seeing both edges in succession.

This lets us think about the issue of orthographics, because we


Figure 6. In the classic system of four humors, the concept of balance is "isonomic" in both the need to find a balance between hot-cold and wetdry but also between the humors themselves, which involves finding a point of symmetrical difference, the "prudence of being sanguine" opposite "prudence with a touch of sentimentality." Melancholy, the "odd humor out," is, on account of the volatility of black bile, the impossibility of fully determining the isonomic point. Thus, the Golden Mean idea: the point that is not a point. Note the arrangement of the quadrated subject within the astrological signs, similar to Ouranos' position in Figure 4. have a right-angle relation to the picture plane that holds our view in place as the trapezoid plays its tricks.

Because the actual flat trapezoid of the Ames window is "already in perspective" although it's on a flat plane orthogonal to our line of view, when we place it perspectivally, the short end will continue to be read as the short end, and we will never think that we are seeing the far edge, only the leading edge (Fig. 5). This makes the rotating window appear to be oscillating side to side, although it's really rotating. ${ }^{14}$ An object placed at a right angle to the window will not oscillate, however, so we will have the uncanny experience of seeing the ruler penetrate the plane of the window as it continues to rotate while the window moves right to left, left to right. All of this happens at a very deep place within our neural networks, a place where the brain's cognitive functions compete with the parasympathetic nerve system to control our sense of what's happening. This immediacy is what makes this illusion able to say something about the expressive function of consciousness, where we attribute motive and intentionality to the object world.

The term isonomic was first used to describe the relation between the two extreme states in the theory of humors. To be healthy and orthographically correct, you had to find the isonomic balance point between hot and cold and wet and dry for the three humors, sanguine, choleric, and phlegmatic. The only problem was with

[^7]melancholy, where even the tiniest bit of black bile was poisonous. This was the odd humor out, but it showed how the isonomic or isomeric point in the other humors was not a balance point but actually a combination or exchange of opposites, each tempering the other, so that, for example, in the case of blood, wet and hot, you needed a bit of the phlegmatic humor to keep your friendliness within bounds, so that you would be prudent while you were falling in love, for example. The isomeric is about the complexity of balance, and the impossibility of defining at as a simplistic kind of division. Melancholy is the key. It teaches the other humors how to behave, but it itself is isolated. It is the teacher who teaches because she can't practice, and when we see her represented, as in the engraving by Dürer (Melencolia $\$ 1$ ), her tools lie on the ground and puzzles of time and space are displayed all around.

## Surprise, Astonishment, the REAL



Figure 7. In Charlie Chaplin's 1928 comic film, The Circus, the Tramp tries to escape the policeman by running onto a turntable being used in an ongoing clown act. The further away Chaplin gets from the space "in front of" the officer, the closer he gets to his back, to the absurd point where he uses his cane to pull the policeman back from his "frontal" chase.

The isomeric/isonomic point is critical whenever popular culture wishes to employ projective geometry to make something funny. In this scene from Chaplin's film, the circus, a chase that begins as linear becomes topological as soon as the tramp and the policeman hop onto a turntable that the clowns had been using as a part of their act. In this new space, being further away from the policeman in the front means being closer to him at the back, so we have a kind of Ames window condition, where the turntable's rotation is like the ruler that goes three hundred and sixty degrees but the chase itself appears to oscillate between the values of closer and further away. The self-intersection of the chase converts the linear model to an alternating current that is non-orientable, and with these two qualities we have, effectively, the definition of the pure projective plane.
Astonishment occurs, when we watch the Ames Window rotate $360^{\circ}$ but appear to waggle from side to side in less than its $180^{\circ}$ range, we are convinced that we are looking at the same edge of the window as it moves from right to left, left to right.

In certain ways, the relation between rotational movement and back-and-forth motion reveals a secret about Lacan's "slide-rule analogy" in Seminar XIV, The Logic of Phantasy. In his use of the ratio of $1 / a$, the interval $a$ is brought within the interval to create the remainder, $a^{2}$. Then, the interval $a 2$ "slides" left across into the $a$ interval to create the remainder $a^{3}$. That slides over to the right again to create $a^{4}$. What is the circular motion that is the Real in contrast to this Imaginary? And, what is the element corresponding to the object protruding through the plane of the Ames
unary trait / recursion


Figure 8. Lacan uses the formula of recursion to define the unary trait's "Fibonacci" properties by, in effect, defining 1 as a "One 1 " (this is called the audioactive property) and relating that to the Fibonacci spiral's origin with a rectangle $1+1$. Recursion continues to produce an evermore precise value of $\emptyset$, the Fibonacci "Golden ratio."
trapezoid? In Lacan's explanation of the meaning of the 1-interval, he explains that the unary trait is grounded in a self-intersecting algorithm that takes the form of $x=1+1 / x$ (Fig. 8). To solve the equation, the "answer" ( $1+1 / \mathrm{x}$ ) must be re-inserted into the "question" ( x - what is it?), and the results produce $2 / 1,3 / 2,5 / 3$, etc. the numbers of the Fibonacci series that, in this "folding," provide successively better approximations of $\emptyset$. The circularity is in the re-insertion strategy that also involves a successive "metonymical" re-use of each number in the series $1,1,2,3,5,8$
.... Self-intersection constitutes a $360^{\circ}$ procedure, with the equivalent happening at the level of $a^{n}$, which is what the viewer of the slide-rule sees, in a face-to-face, $180^{\circ}$ view, in studying this analogy.

Are we justified in saying that Lacan's slide-rule analogy is a version of the Ames Window Illusion? The test would be to show the point where the error of thinking that we see only one edge of the window, oscillating right to left, left to right, is confronted by the presence of an object orthogonal to the plane of the trapezoidal maquette. What is this orthogonal element? Orthos as we know has several meanings. The first is that of correctness. The right angle is orthographic, "correct," because it is the angle of inspection that replicates the angle of production at every point on the surface of demonstration. This allows the orthographic drawing to directly represent measures of lines and angles that appear on the drawing. It is also the factor by which, in parallax vision, allows the brain to correct the curved lines that are actually seen, thanks to the parallax situation, to one the conforms with the "truth" of the straight line in reality, while retaining a psychic memory marker that allows the viewer to reflect on the complexity of the meaning of "straight" and even confront the ineluctable differences between the $360^{\circ}$ "gravity model" and the tensioned line model.

The $90^{\circ}$ meaning of orthos penetrates the circular idea in the same way the line of sight penetrates the orthographic model. It is a "gateway" that is guarded by the two-headed dog, Orthos, because there is no way to resolve the two opposed meanings of straight. In Lacan's sliderule analogy, the counterpart to the ruler sticking through the pane of the Ames Window is (1) the frontality of the slide-rule itself and (2) the viewer of the diagram's mathematical demonstrations, where it seems "incredible" that $a+a^{2}$ should equal 1 , then equally incredible that $a-a^{2}$ should equal $a^{3}$, or then that the procedure, when "waggled" from right to left and left to right, would segregate the odd powers from the even ones. All the while, the 1 as unary trait is "deepening" itself through the algorithm of recursion, $\mathrm{x}=1+1 / \mathrm{x}$. Deepening is equivalent to the parallax perspective of the Ames Window that forces us to see (= demonstration) the "depth
function" of the powers of $a$. The $a$ "extends itself" to a vanishing point on an imagined horizon and stabilizes the space of our frontal, $180^{\circ}$ view.

Our astonishment at Lacan's slide-rule analogy is a convergence and troubled overlap between two logics, the $180^{\circ}$ depth condition, where we are fascinated as we watch the left-right powers of a produce increasingly smaller intervals as each new remainder slides back into its source, and the $360^{\circ}$ (productive) rotation that is the "gravity" that produces the side-to-side experience. I think that here it is appropriate to talk about how the lines that converge at the vanishing point on the horizon is also a kind of fall, and how this fall relates to projective topology.

## Flight in Free-fall

Figure 3 condenses the logic of projective space as an inside-out inversion. The horizon becomes a central point. All of the lines heading for vanishing points on it, at any location, are reproduced in the form of the vectors representing all the related families of parallel lines that meet at those points. What had headed out to the edge of space, to points "lying at infinity," now pass through a point that has repackaged infinity into a single middle position, a kind of reverse "black hole," in that lines radiate out of it rather than disappear into it.

Would it be too colorful to say that, in projective space, every parallax view of lines converging in the infinite distance is really about falling? We get to this point in the case of talking about the core, the raw form of representation, what Freud called Vorstellungsrepräzentanz. This is when a child acts out by using representation in illegitimate ways, saying "the dog goes meow" and "the cat goes bow-wow." The Rat Man for example called his father "a lamp, a towel, a plate" as an insult. And, Lacan cited the phrase "Colorless green ideas sleep furiously," in Seminar XX, Crucial Problems for Psychoanalysis. Acting out in such obvious ways erroneously gives the impression that you have to be, as his father called the Rat Man, "either a criminal or a genius." Vorstellungsrepräzentanz is more basic, more universal. ${ }^{15}$ As John Hendrix has put it, Freud cited the moment of transfer from the unconscious to the conscious mind, calling it an act. In history, this had been represented as the necessary bond between sensible images and words (language as necessary to perception and vice versa). The Vorstellungsrepräzentanz is the presentation of the word in what Cassirer called its "expressive function" (Ausdrucksfonktion), a level of pure astonishment and surprise, before intellection has had the chance to domesticate it into something relational or cognitive.

We can experience this level at the point in the rotation of the Ames Window when we, the viewers, mistake the edge of the thin maquette as the same edge, even though the maquette rotates $360^{\circ}$, showing us both of its ends. Thanks to its forced perspective and our neural network's "need to believe" in the representation of the parallax that holds it in place as a representation, the one edge that is really two edges oscillates back and forth, left to right, right to left. This reality casts

[^8]the feature that actually seems to rotate $360^{\circ}$ into the role of a fictional, magic object. In Rat Man terms, the father has become the lamp, towel, or plate that is ridiculous, magical, obscene.

By switching the forced perspective magic into reality, the representation of reality becomes magic. Our sense of stereognosis, so eager to designate the world as a matter of handedness, is so powerful that anything standing in its way is made to appear as a miracle, and here we have the science of divination in a nutshell. The sun does not disappear at sunset and re-appear the next morning, it becomes a new sun with each dawn, because our sense of chirality, our parallax, has converted $360^{\circ}$ into $180^{\circ}$, rotation into a face-to-face, presentational stereognosis. The Ames Mirror has been able to put this "mythic" state of mind directly before us because it has been able to solicit our neural network's preference for stereognosis, for parallax. Its demand for chiral order throws what is ordinary reality into reverse gear, and we see the ruler passing magically through the window panes.


Figure 9. Paul Newman and Robert Redford in The Sting (1973). Henry Gondorff (Newman) and Johnny Hooker (Redford) are the Con and Shill respectively. The Con ("confidence man") is known to be corrupt and suspected by the Mark; the Shill, however, appears to be coercible. He presents himself as betraying his boss in order to convince the Mark that he has revealed the "inner workings" of the Con that the Mark can now use to advantage. Of course the reverse is true. A larger Real has been constructed around the faked "reality."

What has resisted the Symbolic? In Lacan's RSI schema, the Real, Symbolic, Imaginary, the Real resists. The Real, Lacan says in Seminar XIV, The Logic of Phantasy, is to be equated with topology. What we call reality is the order the Symbolic hastens to construct around any unknown thing. In the 1973 film The Sting, two professional con-men (Robert Redford and Paul Newman) construct a fake gambling operation aimed to deceive a single villain, the Irishman Doyle Lonnegan in retribution for his murder of their kindly con-artist friend, Luther Coleman. A betting parlor is erected in a back alley; actors are hired to play both staff and clientele. Lonnegan is told that the parlor can give favored clients an inside tip because a crooked accomplice at the Wells Fargo wire service delays the results of races so that certain horses can be rebroadcast to win, place, or show for the clientele who believe that the radio broadcast is happening in real time.

Think of the sting of The Sting as reversed entropy, or (more vividly) a fallen glass of liquid being re-assembled when the film is run backwards. Lonnegan is made to believe in the perspectival reality of the fake betting shop, its chiral opposition of cashier-bankers to betting clients, thanks to the fact that Redford's character, Johnny Hooker, has convinced him that he has turned against his boss and, as betrayer, certified the authenticity of the plan to bring the Con, Henry Gondorff (Newman) down.

The Ames Window uses the structure of the confidence trick to demonstrate how, in human perception in general, the Vorstellungsrepräzentanz is ever active, ever ready, ever in place -
neurally - to again stage a small demonstration of the expressive function, where what is ordinary will appear as magical and what is fake will be held in place by parallax. What we have theoretically in this demonstration is what Giambattista Vico called a "proof of the body," where by renouncing the style of Aristotelian predication a new kind of syllogism, still true to the logic of major and minor premises connected by a middle term, engages a neural mandate simply by silencing the middle term. The confidence trick does the same by the construct of the shill; the Ames Window does the same by the theatrical function of the maquette flat window, whose two edges appear to be the same edge, always in front, with a further edge at the rear. Like the twins Castor and Pollux, taking turns being alive and dead, we accept the mandate because we believe we are on the side of the living, that what vanishes must vanish into the "death" at the point on the distant horizon. The twins seem to be the same - after all, this is the definition of twins - and the ordinary object penetrating their "window pane" appears as a miracle. This is the deal struck with the gods of Hades, forbidding the twins to meet ever again; but when they do, as in the story of Simonides, a miracle happens. ${ }^{16}$ The poet narrowly avoids death thanks to the "two strangers" who call him outside - outside of what we have to ask? Outside of the chiral face-to-face exercise of matching names to places, Simonides returns to a re-match, corpses returned to their places and, hence, names and afforded proper burials.

The contrariwise miracles of the two intersecting parallaxes is hard-wired, neurologically, but the circuits of that those networks is simultaneously event and word; a brain without language, a brain without an unconscious, would not appreciate the Ames Window experiment. There would be no Vorstellungsrepräzentanz and no expressive function, no overlap, no stereognosis. These owe to the cut that is the mirror function that structures space so that when we look at the world, the world looks back. The eye that looks back has no specific position; it is generically orthogonal, its two components of correction and $90^{\circ}$-at-all-points stare back wherever there is lack, which is every where - human space. When Freud wrote his cryptic note in 1938, "Psyche is extended, knows nothing of it," he anticipated Lacan's elaboration of the Psyche's Real as a topology describing that extension as non-orientation and self-intersection. ${ }^{17}$

[^9]
[^0]:    ${ }^{1}$ Although there is no direct etymological relation between chase and chastity, the chemical process of removing excess dye from a fabric or coloration from a metal surface before embossing suggests some overlap from the ideas of flight and purity in the Apollo-Daphne story to more general understanding of the relations of the shot beam to the horizonal sphere.

[^1]:    ${ }^{3} \mathrm{~A}$ contronym is non-oriented and self-intersecting; therefore it is a verbal counterpart to topological figures in projective space.

[^2]:    ${ }^{4}$ It is surprising to find out that virtually no Lacanians make reference to this aspect of projective geometry, concentrating instead on the results of this inside-out spatial inversion, toruses, cross-caps, Möbius bands, etc. But, these are the immersions of the true projective topologies, represented more accurately by the "fundamental polygons" that are like folding instructions. Myths follow the folding instructions, also the man and woman follow them. Topology is not just a "pure mathematics" but the ethnological principles of motion, stillness, repetition (hence, ritual), boundaries, death, transitions (liminal), etc. This is "lived topology," and we can understand it by knowing about its mathematical forms as long as we don't think of mathematics as the only reality of projective space. Lacan says this directly, and in many places; and it is evident even when he doesn't, as in the Three Prisoners' Dilemma he discovers in 1935, long before he gives the official date to his topological interests, 1961. This is Lacan's general interest in the EFQ, the ex falso from which all things may spring forth, in purity and truth. Thanks to Iraj Esmaeilpour Ghoochani for bringing up the relation of truth to "the pure flow."

[^3]:    ${ }^{5}$ Chat GPT: "Tiresias was a blind prophet who lived in Thebes, a city in ancient Greece. He was said to have been the son of a shepherd named Everes and a nymph named Chariclo. Chariclo was a daughter of the god Oceanus and the goddess Tethys, making Tiresias a grandson of two powerful deities." Cadmus and Tiresias were both descended from the Semitic tribes that moved up from the Syrian plains to Cadmeia in Caria. Cadmus is a Semitic word meaning "eastern." As Barkan notes, they were a race said to be "sewn from the earth," harvested by the planting of dragon's teeth, and so Tiresias, in his encounter with copulating snakes, was in effect transformed into the metonymy of his own origins, a contronym that today loans its powers to the medical profession in the form of the caduceus, used by Hermes to cross boundaries as a herald protected from prosecution to the point that he could cross the boundaries between life and death. See Leonard Barkan, "Diana and Actaeon: The Myth as Synthesis," English Literary Renaissance 10, No. 3, Studies in Shakespeare (Autumn 1980): 317-359. Also: Robert Graves, "Europe and Cadmus," The Greek Myths 1 (New York: George Braziller, 1955), 197.

[^4]:    ${ }^{6}$ Vico: Saturn "gets his name from sati, 'sown' [fields]." The famous anagram square, ROTAS/SATOR/TENET/ OPERA/AREPO, whose mystery has not been fully mastered, is evidence that the enigma of retroactive self-origins has perplexed generations since early days. Giambattista Vico, The New Science of Giambattista Vico, trans. Thomas Goddard Bergin and Max Harold Fisch (Ithaca: Cornell University, 1948), $\$ 3$.

[^5]:    ${ }^{11}$ Where signals are all about hunting or being hunted, amplification is necessary. Again, the figure-ground distinction is critical. Studies of crayfish in ponds, predator turtles are detectable because of small follicles able to use the stochastic resonance of pond's white noise to amplify the weak signals of the turtle's movement. The white-noise becomes the ground against which the signal is "pushed forward" as detectable. In the same way, perspectival figureground distinction amplifies the nearness of the figure and the distance of the ground, formalizing it thanks to "stochastic" procedures that create a perceptual blanket to generalize distant objects, melding them into a single ground. Parallax, as anyone who has had to use a camera knows, makes focus easier for distant objects, difficult for ones in the near field. Anything in the distance of importance must be "amplified" and brought closer by distinguishing it from a general blended field.
    ${ }^{12}$ In effect, the 1 of $Y$ is the "depth" of the projective plane as a whole, at (literally) the position of a unit. Depth in 3-d is sagittal, but in 2-d it is the chase around the topological surface, such as that made by Apollo of Daphne, leading to the two properties of projectivity, Daphne's self-intersection (paralysis) as a laurel tree, and Apollo's desire as contronymic to Daphne's hate, although they were shot contronymically at the same time, possibly by the same arrow (non-orientation). The contronymic chase is also the theme of the "Appointment in Samarra" story group, where the servant attempting to flee death runs straight into him, "right on time": temporal as well as spatial coincidence. The other aspect of the uncanny, the subject who forgets that he/she is dead, is the complementary theme of selfintersection, as this interval is portrayed as the Thesean labyrinth, folds of a single passageway that equalize outside and inside with a fractal algorithm that doesn't change although it generates continuous change.

[^6]:    ${ }^{13}$ Cathesis exists also for figures in 2-d projective space, but it is converted to the theme of the (contronymic) chase. Just as the vanishing point seems to follow the traveler as he/she moves across the landscape, the fleeing subject can never escape the object of fear in 2-space, since the vanishing point is the product of the viewing point, both distant and close with any attempt to escape. The desire to escape produces cathesis immediately, suggesting that metaphoric suppression directly produces metonymic expression. In effect, this is the space of the trap. Demonstrations of the Ames Window Illusion abound; this is one: https://nerdist.com/article/ames-window-illusion-how-does-it-work/

[^7]:    ${ }^{14}$ The Ames Window illusion was invented by Adelbert Ames Jr. (1880-1955) in 1946. Ames pioneered the study of physiological optics at Dartmouth University. As a student at Harvard, he studied under George Santayana and William James.

[^8]:    ${ }^{15}$ John Shannon Hendrix, "The Vorstellungsrepräzentanz," Vestigia, International Network of Psychoanalytic Practices 3, Issue 1 (December 12, 2022). https://inppjournal.org.uk/wp-content/uploads/2021/12/John-Shannon-Hendrixarticle.Vestigia.V31.pdf

[^9]:    ${ }^{16}$ Don Kunze, "Cloud Nine: A Lover's Guide," in Ceilings and Dreams, the Architecture of Levity, ed. Jodi LaCoe and Federica Goffi (New York and London: Routledge, 2019).
    ${ }^{17}$ Sigmund Freud, (1938) "Findings, Ideas, Problems," The Standard Edition of the Complete Psychological Works of Sigmund Freud 23: 299-300.

