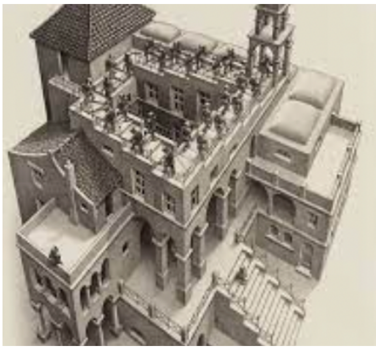
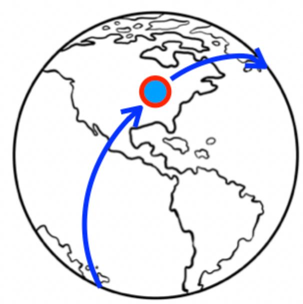
What Do "Self-Intersecting" and "Non-Orientable" Mean?

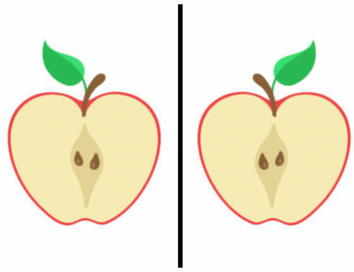


Any understanding of projective geometry ("topology" for Lacan) requires an understanding of two terms: self-intersection and non-orientation. Every topological form (Möbius band, cross-cap, torus, Klein bottle, Boys Surface, etc.) is self-intersecting and non-orientable. What does this mean? "Non-orientable" is equivalent to Lacan's Real. We cannot show non-orientation without creating a visual illusion or paradox, such as the Necker Cube or Ames Window. We must "show the impossible" in the fashion of the Dutch artist, M. C. Escher, who drew staircases that seemed to go up and down at the same time, or create devils out of the space in between angels.

Self-intersection may seem to be an easier concept to digest, but it too involves a difficult peculiar to the "real projective plane" that is the basis of projective geometry. It is a circuit that is able to move energy from one place to another, but with the principle of Newton's Law about conservation. Nothing is gained or lost, meaning that the system operates as a kind of Leibnizian monad. In geometric terms, this means that there is not simply a "continent" barrier keeping the inside from the outside and *vice versa*, there simply is not a boundary. There is no line drawn around an area, as there would be in Euclidean space. Imagine a point on the 3-d globe of the earth. How "distant from itself" — how much space is there, traveling out from any/all directions — to travel to return to the point of origin? The answer is rather easy: the distance is the circumference of the earth, minus any circumference of the point itself, which we presume to be zero.



always 360º circumference ("great circle")



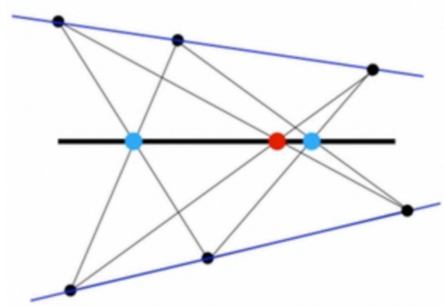
mirroring is a cut, not a reflection

(when a cut has this effect elsewhere, it is called "katagraphic")

If we move to the case of 2-d projective space, the answer is more perplexing. First, when we travel around projective geometry's equivalent of a sphere (an "a-sphere") we arrive back at the starting point *in a reversed position*. We leave with our back to the origin point and arrive facing the origin point in Euclidean space, but in projective space we return from where we left face to face with our departing self. In other words, an a-sphere is the condition of mirroring, a "cut" between a reflecting and reflected space, *at every point within the space*. Because a mirror is, counter to what most people believe, not a reflection but rather a slice through space that creates two "chiralistic" (left-right different) faces, this means that the a-sphere is, if we measure it at any and all points, essentially a cut.

It may seem difficult to think about any space as a cut, instead of a flow of a volume, a "room." But, if we remember that projective space is self-intersection *and* non-oriented, this is the conclusion we must draw. We must remember that the combination of these two qualities is the *sine qua non* of projective geometry, that without them we are talking about something else.

This is the fallacy of the many authors who believe that Lacan's topology begins with the "Königsburg Bridge Problem," which was settled by the mathematician Leonhard Euler in 1735. The seven bridges cannot be all crossed, only once each. This is a problem resolved by graph theory. It is not a even a case of Affine Geometry (Euclid without measures of distances or angles), called "rubber-sheet geometry." The standard authority cited by authors who give the Königsburg Bridge Problem as the origin or projective geometry, Jeanne Granon-Lafont, is doubly wrong. Projective geometry began with, as Lacan himself cites in several places, with the theorems of Pappus of Alexandria, around 300 c.e. These were rediscovered by the architect Girard Desargues and Blaise Pascal in the 17c.



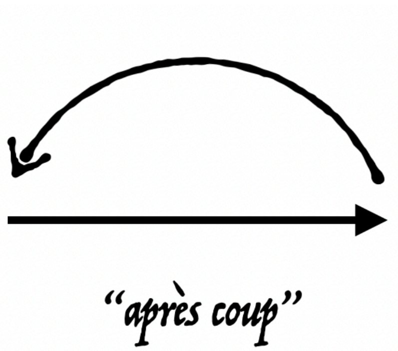
Pappus' Theorem

Pappus' theorem is easy to reproduce. The involve the calculation of points that are "co-linear" (lying along a single straight line), made by connecting six points located anywhere on two lines placed at any angle to each other. This "anywhere-everywhere" freedom of the lines and points means that, in the space of the lines and points there is *generally and universally* a co-linearity. The points that intersect specifically represent points that are, *generally and universally*, self-intersecting. The *method* by which they intersect involves a twist. For each pair of points to connect, A and B on one line must be connected in reverse order: B' then A'. With AB:B'A' we have the same "slice" that the mirror makes with its katagraphic cut. In other words, Pappus "is saying" that (projective) *space itself* is self-intersecting and non-orientable.

This is not the space of Euclid. To prove Euclid's theorems, we require a straight-edge and compass. We need a space outside of the drawing to construct it. Pappus' projective space does not — can not — distinguish between a viewer and the viewed. It has no way to give space an extra dimension to construct and describe it. It is *self*-intersecting, in a way that compares to the Cretan Liar who cannot say anything outside of his status as a Cretan. He says "All Cretans are liars," but this very statement is inside the space of the claim, so it negates itself; but if it negates the fact that all Cretans are liars, then the Cretan is *now* telling the truth. We have a space created by the claim that automatically implies a *time*, where truth and lies alternate. This space of self-intersection is also a time, an alternation of True and False, Yes and No, Alive and Dead. It is a space-time of non-orientation.

The Bridges of Königsburg are neither self-intersecting (in fact, they fail at this, as Euler proved), nor are they non-oriented. Instead of being an issue of left and right, as in the case of the "chiralistic" faces of space cut by a mirror, they are a matter of odd and even. An odd number of bridges cannot be all crossed only once, but an even number can. Since this graph theory problem has nothing to do with the *rule* of self-intersection (half of its cases, the odd ones, cannot accomplish self-intersection) and there is nothing non-orientable about a journey over them, this problem cannot be logically or historically the origin of topology that many if not most Lacanian writers say it is.

Lacan not only understands the rules of non-orientation and self-intersection, he knows and cites the authors of these rules: Pappus and Desargues. He knows and cites the katagraphic cut, in the story of the "Injunction of Popilius" in Seminar IX, *Identification*. But, even more conclusively, he bases his entire notion of the space-time of the human subject in relation to the world that begins with the Mirror Stage, Lacan's theoretical and historical origin, his first demonstration of the radical role of the Other in psychoanalysis.

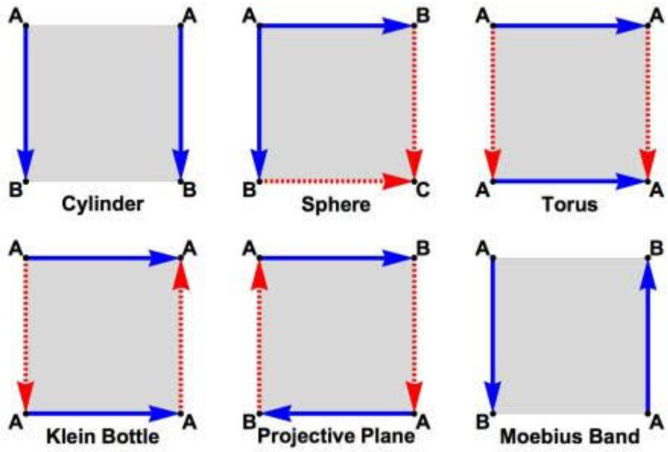


From this point on, Lacan is able to show the relation of "specular" space to the signifier and, in particular, language's own case of non-orientation: the effect of retroaction — *après coup* — whereby all meaning depends on the case of the double temporality of the unit of meaning, the sentence. To know what a sentence means, we must *wait for* the ending. The end of a sentence, which by definition exists in a temporality it creates, must relate to the beginning; at the same time, the beginning is re-constructed by the meaning we acquire only at the end.

In literature, this rule is played out by whole stories, where the ending is required to "answer to" the beginning. We have topology from the moment Lacan realizes this double flow as the essence of language, and language as a *flow* of signifiers in two directions. These flows take time and are sequential, but the meaning they create is a *unity* of these opposed, non-orientable motions. While the sentence itself is a linguistic case of self-intersection (beginning to end, then end to beginning), it also produces a "cost" of non-orientation. This is a cost for theory, not language itself. We cannot speak and make sense without non-orientation, but it is difficult for us to understand *why* this is the case. We cannot explain contradiction; we can only show how it works to construct everyday reality.

The Fundamental Polygon

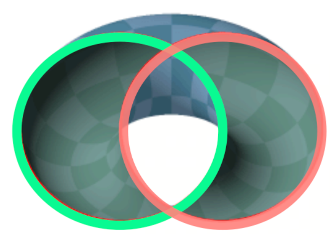
In order to theorize self-intersection and non-orientation, we must create a space of demonstration, where we can at least pretend to "operate on" the topological Real. This is called "immersion." The donut, bagel, or bicycle tire we see and call examples of the torus *are not the projective form of the torus*. They are immersions. The 2-d projective torus has been given an extra dimension, a line or vector by which we can describe and define it. The only way to describe/



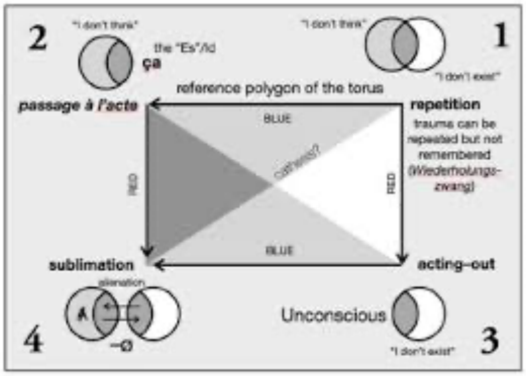
define a projective figure without adding a new extra dimension is to construct what is called the "fundamental polygon." This is a standard graphic device that mathematicians use to differentiate various topological forms.

The example of the Möbius band is the easiest to follow. We see that two "edges" are joined, but first there must be a twist: AB must be connected to BA. There is a temptation at this point to think of the fundamental polygon as a sheet of paper with edges and corners. Many of not most Lacanians make this mistake. The fundamental polygon is not a sheet of paper requiring a third dimension of space in which the paper can be twisted or folded. It is a 2-d surface that remains in 2-space. When a red and blue vector diverge from A in the example of the torus, they diverge *simultaneously*. Even though a torus that has been immersed into 3-space looks as if the blue vector are folded together, then the ends of this tube are connected, this does not produce a form that is non-orientable. It is not the torus as a projective form, but a torus that is immersed: a donut, bagel, or bicycle tire. All of these have *volumes*; all of them can be *cut*.

However, it is possible to use cuts to return to the torus's projective nature. It is possible to cut a bagel into two pieces and, while the knife makes a 360º circuit (self-intersection) it also twists 180º. The result is that the two halve have Möbius-band surfaces that face each other. Like the mirror, they cut space into left and right halves, but each half "claims" to be a full 360º, the real world and its mirror double. This Möbius-bagel can hold twice as much cream cheese! But, on account of the two Möbius band faces, we now have a topology that is non-orientable.



Another cut we can make into an immersed torus is instructive. This is the "Villarceau cut," a diagonal that produces a section where we can clearly see two intersecting circles. If we realize that the two circles "reach over" into opposite parts of the torus, we can say that they are "Euler circles" rather than simple graphic circles. The space of their overlap is a void. Lacan uses Euler circles to define the corners of the fundamental polygon of the torus that he uses, in Seminar XIV (*Identification*) to describe the relation of repetition to sublation, related to demand and desire. Lacan is not using the fundamental polygon as "folding instructions" for a sheet of paper, but rather is seeing that the blue-red vectors *diverge simultaneously* from the corner marked "repetition" to define antipodal cases of the *passage à l'acte* (the outside of the Symbolic) *versus* "acting-out" (the inside of the Symbolic). Then the vectors simultaneously *converge* on the corner identified as sublation, which Lacan has elsewhere connected to metaphor's logic of substituting



The fundamental polygon of the torus

one signifier for another. Although Lacan himself say that his topological analysis properly began with Seminar IX, his topological *theorizing/thinking* began much earlier — at least as early as his thesis of the Mirror Stage!

In this one diagram, Lacan relates the topology of the torus to the broad features of subjectivity: the relation of demand to desire, the central function of metaphor/metonymy, the position of the Unconscious in relation to the Es, the controversial corners of his L-Schema. All of these are construed as a dynamic interaction of *flows of energy*, with properties such as turbulence and laminar alignment. Lacan is not only original in his topological thinking, he stands alone in his assertion that topology is the Real of the subject. But, even more astounding is his claim that it is possible to *theorize* this Real thanks to our ability to access the non-immersed logic of projective forms.

To conclude this short essay on non-orientation and self-intersection, let me summarize points that need to be elaborated later:

- Self-intersection is about the completion of *circuits* that conserve but move energy. This answers Freud's call for a "scientific psychology," made in 1895; but Lacan is able to factor in the effect of a Death Drive and treat it as a "master drive" whose logic lies behind the other drives.
- Non-orientation is about the incommensurability of projective topology with Euclidean representational space. This requires us to use the fundamental polygons of projective surfaces in theoretical ways, by finding parallels in the reality of the subject, clinical, historical, and in works of cultural production (art, architecture, cinema, fiction, etc.) that provide speculative and experimental conditions for testing what is essentially resistant to any paraphrase or thesis.
- Any advance of Lacan's legacy requires a correct understanding of topology. This mandates a theoretical acceptance of self-intersection and non-orientation as *foundational*. Theoretical progress and theoretical failure amount to the same thing, in that both are grounded in the *production of jouissance*. Inasmuch as theory cannot jettison its aspect as representation, failure is built-in. This is not a deterrence but rather the basis for a positive program.
- Psychoanalysis's greatest achievements lie in conditions where "Euclidean" time and space are negated, folded, or twisted: the Death Drive, dreams, jokes, the Unconscious. In these areas, the Unary Trait plays indispensable roles. Topology is inseparable from the Unary Trait, which is also self-intersecting and non-orientable.